

Algebra + Advanced Maths

MATHS ORVJARE SHORT BOOKS NOTES

(For competitive exams)

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Number System (संख्या पद्धति)

Classification of Number:-

Complex Number (समिश्र संख्या)

Real Number (वास्तविक)

(Can be denoted on number line)

Ex:- 3, -5, 5, $\frac{16}{3}$, 0.06, $\sqrt{7}$

Imaginary Number (काल्पनिक)

(can't be denoted on number line)

Ex:- $\sqrt{-7}$, $\sqrt{-10}$, $\sqrt{-3}$ ----

Rational Number (which can be written in $\frac{p}{q}$ form $q \neq 0$ and p, q are integers)

Ex:- $\frac{7}{3}$, $\frac{11}{2}$, $-\frac{8}{3}$, $\frac{19}{6}$, $\sqrt{9}$ ----

Irrational Number (which can't be written in $\frac{p}{q}$ form)

Ex:- 0.13426137....., $\sqrt{2}$, $\sqrt{3}$, $\pi = 3.141592$ ----

Decimal Number

• Terminating decimal

$$0.5 = \frac{1}{2}$$

$$\frac{63}{100} = 0.63$$

• Non-terminating repeating decimal

$$\frac{1}{3} = 0.333\ldots$$

$$\frac{56}{99} = 0.565656\ldots$$

• Non-terminating Non-Repeating

$$\sqrt{2} = 1.414\ldots$$

↓
Irrational Number

Rational Number

Integer (पूर्णांक)

Negative integers
(-1, -2, -3, ..., ∞)

Non-negative integers
(0, 1, 2, 3, ..., ∞)

Whole numbers
(पूर्ण संख्या)

Odd number (विषम सं.)
(Not divided by 2)

Even number (सम सं.)
(divided by 2)

Natural Numbers (प्राकृत संख्या)

Prime Number
(has only 2 factors)
(2, 3, 5, 7, 11, ...)
↳ Smallest Prime No.

Neither Prime
nor composite
1

Composite
(have more than 2 factors)
(4, 6, 8, 9, 10, 12, ...)

	Prime No.	Sum
1 - 25	9	100
1 - 50	15	328
1 - 75	21	712
1 - 100	25	1060

- Each Prime number can be written in $(6K \pm 1)$ form but every $(6K \pm 1)$ form may not be necessarily prime number.

Eg. $\rightarrow 13 (6 \times 2 + 1)$ [Prime no.]

Eg. $\rightarrow 25 (6 \times 4 + 1)$ [not prime]

- Composite Numbers:- More than 2 factors.

Eg. $\rightarrow 4, 6, 8, 9, \dots$

1 → Neither prime nor composite

2 → Only Even Prime number & Smallest Prime number

3, 5, 7 → Only pair of consecutive odd Prime number

4 → Smallest composite number.

9 → Smallest odd composite number.

- Relatively Prime/co-prime number :- Two numbers having $HCF = 1$ are co-prime numbers.

Eg. $\rightarrow (2, 3), (16, 9), (11, 3)$ etc.

- Twin-prime number :- Two Prime number with a gap of 2.

eg. $\rightarrow (3, 5), (5, 7), (11, 13)$ etc.

- Perfect numbers :- If the sum of all the factors (excluding that number) is equal to that number.

Eg. $\rightarrow 6 \rightarrow (1, 2, 3)$
 $\quad\quad\quad + \quad + \Rightarrow 6$

- Only pair of prime no. with a gap of 2 is 3, 5, 7.

- Smallest 3 digit prime $\rightarrow 101$

- Largest 3 digit Prime $\rightarrow 997$

Divisibility Rule

- 1 is not divisible by any number except 1 but 1 is a universal factor.

- Divisibility Rule of 2, 4, 8, 16:-

- 2 → Last digit should be divisible by 2.
- 4 → Last 2 digits should be divisible by 4.
- 8 → Last 3 digits should be divisible by 8.
- 16 → Last 4 digits should be divisible by 16.

- Divisibility of 3 and 9:-

- 3 → Sum of digits should be divisible by 3.
- 9 → Sum of digits should be divisible by 9.

- Divisibility Rule of 5, 25, 125:-

- 5 → Last digit should be 0 or 5.
- 25 → Last 2 digits should be divisible by 25.
- 125 → Last 3 digits should be divisible by 125.

- Divisibility Rule of 6:-

- 6 → $6 = (2 \times 3)$ (co-prime factors)

If a number is divisible by 2 & 3 both, that number will also be divisible by 6.

- Divisibility Rule of 7, 11, 13:-

- Make pair of 3 digits from RHS
- Add alternate pairs & take difference
- If difference is divisible by 7, 11, 13 then number will be divisible by 7, 11, 13 respectively.

Eg. $\rightarrow 5922$

$$\underline{005922} \rightarrow 922 - 5 = 917$$

$7 \rightarrow (\checkmark)$, $11 \rightarrow (x)$, $13 \rightarrow (x)$

Eg. $\rightarrow 14885$

$$\underline{014885} \rightarrow 885 - 14 = 871$$

$7 \rightarrow (x)$, $11 \rightarrow (x)$, $13 \rightarrow (\checkmark)$

Eg. $\rightarrow 380247$

$$\underline{380247} \rightarrow 380 - 247 = 133$$

$7 \rightarrow (\checkmark)$, $11 \rightarrow (x)$, $13 \rightarrow (x)$

• $ABAB \rightarrow$ divisible by 101

$$\text{Eg. } \rightarrow 73 \times 101 = 7373$$

• $ABCA BC \rightarrow$ divisible by 1001

$$\text{Eg. } \rightarrow 687 \times 1001 = 687687$$

• $7 \times 11 \times 13 = 1001$

• Divisibility Rule of 11:-

If the difference between the sum of the digits at odd places and sum of the digits at even places is zero or multiple of 11.

$$\text{Eg. } \rightarrow \underline{1}6\underline{6}4\underline{5}2$$

$$1 + 6 + 5 = 12$$

$$6 + 4 + 2 = 12$$

} difference = 0 (number is divisible by 11)

$$\text{Eg. } \rightarrow \underline{7}9\underline{4}5\underline{9}3\underline{8}$$

$$28 - 17 = 11 \text{ (Number is divisible by 11)}$$

• Divisibility Rule of 12:-

If a number is divisible by 4 and 3 both then that number will also be divisible by 12.

Remainder (शेषफल)

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

भाज्य भाजक भागफल शेषफल

$$\boxed{D = Qd + r} \quad d > r \geq 0$$

- +ve Rem. + -ve Rem. = Divisor

Eg. $\rightarrow \frac{62 \times 102}{14}$, Remainder = ?

$\frac{62}{14}$, Remainder = 6, $\frac{102}{14}$, Remainder = 4

$\frac{6 \times 4}{14} = \frac{24}{14} \rightarrow \text{Remainder} = 10$

- Concept of Remained:-

$$\frac{A+B+C+D}{M} \rightarrow \text{Remainder} = \frac{A_R + B_R + C_R + D_R}{M}$$

$$\frac{A \times B \times C \times D}{M} \rightarrow \text{Remainder} = \frac{A_R \times B_R \times C_R \times D_R}{M}$$

- Concept of Negative remainder:-

$$\begin{array}{r} 11 \overline{) 9718} \\ 88 \\ \hline 9 \end{array}$$

Positive remainder

$$\begin{array}{r} 11 \overline{) 9719} \\ 99 \\ \hline -2 \end{array}$$

Negative Remainder

$-2 + 11 = 9 \rightarrow \text{Remainder}$

- $\frac{X}{D} \rightarrow \text{Remainder} = R$, then $\frac{2X}{D} \rightarrow \text{Remainder} = \frac{2R}{D}$

- $\frac{X}{D} \rightarrow \text{Remainder} = R$, then $\frac{X^2}{D} \rightarrow \text{Remainder} = \frac{R^2}{D}$

• If $\frac{4^n}{6}$, then always remainder = 4

• Fermat's theorem:-

$$\frac{a^{p-1}}{p}, \text{ Remainder} = 1$$

Where a, p are co-prime numbers.

Eg. $\rightarrow$ Find the remainder in $\frac{2^{100}}{101}$?

$$\frac{2^{100}}{101} \rightarrow R = 1.$$

• Wilson theorem:-

$$\frac{(p-1)!}{p}, \text{ Remainder} = (p-1)$$

p is a prime number.

Eg. $\rightarrow$ Find the remainder in $\frac{28!}{29}$?

$$\text{Remainder} = 29 - 1 = 28$$

• Remainders of algebraic expressions:-

$$\frac{(x+a)^n}{x} \rightarrow R = a^n$$

OR

$$\frac{(tx+a)^n}{x} \rightarrow R = a^n \quad [t = \text{multiple of } x]$$

$$\frac{(ax+1)^n}{a} \rightarrow R = 1^n = 1$$

$$\frac{(ax-k)^n}{a} \rightarrow R = (-k)^n$$

$$\frac{(ax-1)^n}{a} \rightarrow R = (-1)^n$$

No. of the form	Divided by $(a+b)$	Divided by $(a-b)$
$(a^n + b^n)$ ($n \rightarrow \text{odd}$)	✓	✗
$(a^n + b^n)$ ($n \rightarrow \text{even}$)	✗	✗
$(a^n - b^n)$ ($n \rightarrow \text{odd}$)	✗	✓
$(a^n - b^n)$ ($n \rightarrow \text{even}$)	✓	✓

- If power is odd,
 $a^n + b^n + c^n + d^n$ is divisible by $(a+b+c+d)$
- If $\frac{10^n}{6} \rightarrow$ Remainder always = 4

Number of factors

$2^p a^q b^r c^s$ (a, b, c are prime numbers)

- Total numbers of factors = $(p+1)(q+1)(r+1)(s+1)$
- Odd (विषम) factors = $(q+1)(r+1)(s+1)$
- Even (सम) factors = $p(q+1)(r+1)(s+1)$
- Prime (अभाज्य) factors = $p + q + r + s$
- Sum of factors $(a^p b^q c^r) = \frac{(a^0 + a^1 + \dots + a^p)(b^0 + b^1 + \dots + b^q)(c^0 + c^1 + \dots + c^r)}{(c^0 + c^1 + \dots + c^r)}$

Ex:- $100 = 2^2 \times 5^2$

$\Rightarrow (2^0 + 2^1 + 2^2)(5^0 + 5^1 + 5^2) \Rightarrow 7 \times 31 \Rightarrow 217$

- Sum of even factors $(2^p b^q c^r) = \frac{(2^1 + 2^2 + \dots + 2^p)(b^0 + b^1 + \dots + b^q)(c^0 + c^1 + \dots + c^r)}{(c^0 + c^1 + \dots + c^r)}$
- Sum of odd factors $(2^p b^q c^r) = 2^0(b^0 + b^1 + \dots + b^q)(c^0 + c^1 + \dots + c^r)$

Number of zero

- Find number of 5 and 2.

Because $5 \times 2 = 10$

Ex. $1 \times 2 \times 3 \times \dots \times 100$, number of zero = ?

$$\begin{array}{r|l} 2 & 100 \\ \hline 2 & 50 \\ 2 & 25 \\ 2 & 12 \\ 2 & 6 \\ 2 & 3 \\ \hline & 1 \end{array} \quad \left. \vphantom{\begin{array}{r|l} 2 & 100 \\ \hline 2 & 50 \\ 2 & 25 \\ 2 & 12 \\ 2 & 6 \\ 2 & 3 \\ \hline & 1 \end{array}} \right\} + \Rightarrow 97$$

So, 2^{97}

$$\begin{array}{r|l} 5 & 100 \\ \hline 5 & 20 \\ & 4 \end{array} \quad \left. \vphantom{\begin{array}{r|l} 5 & 100 \\ \hline 5 & 20 \\ & 4 \end{array}} \right\} + \Rightarrow 24$$

So, 5^{24}

No. of zero = 24 (Minimum power)

Eg. $\rightarrow 1 \times 2 \times 3 \times \dots \times 150$, number of zero = ?

$$\begin{array}{r|l} 5 & 150 \\ \hline 5 & 30 \\ 5 & 6 \\ \hline & 1 \end{array} \quad \left. \vphantom{\begin{array}{r|l} 5 & 150 \\ \hline 5 & 30 \\ 5 & 6 \\ \hline & 1 \end{array}} \right\} + \Rightarrow 37$$

5^{37}

No. of zero = 37

Eg. $\rightarrow$ No. of zero in the product of $5 \times 10 \times 25 \times 40 \times 50 \times 55 \times 65$
 $\times 125 \times 80$

$$\Rightarrow 5 \times 10 \times 25 \times 40 \times 50 \times 55 \times 65 \times 125 \times 80$$

$$= 5 \times 2 \times 5 \times 5^2 \times 2^3 \times 5 \times 5^2 \times 2 \times 11 \times 5 \times 13 \times 5 \times 5^3 \times 2^4 \times 5$$

$$= 5^{13} \times 2^9 \times 11 \times 13$$

$$\Rightarrow (5 \times 2)^9 \times 5^4 \times 11 \times 13$$

No. of zero = 9

Unit Digit (इकाई अंक)

Type-1:- Addition $\rightarrow$ Add only last digits

Ex:- $5463 + 2812 + 31108$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow$
3 2 8

+ +

$\Rightarrow 13 \rightarrow \text{last digit}$

Type-2:- Multiply $\rightarrow$ Multiply last digits

- $5 \times \text{odd} \rightarrow \text{U.D.} \rightarrow 5$
- $5 \times \text{even} \rightarrow \text{U.D.} \rightarrow 0$

Ex:- $353 \times 179 \times 127 \times 202$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow$
3 $\times$ 9 $\times$ 7 $\times$ 2

$\Rightarrow 8 \rightarrow \text{Unit digit}$

Ex:- $35 \times 71 \times 132 \times 453$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow$
5 1 2 3

$\times$

$\Rightarrow 0 \rightarrow \text{Unit digit}$

Type-3:- Power

- If U.D. $\rightarrow 0, 1, 5, 6 \rightarrow$ Never change

Eg. $\rightarrow$ U.D. of $(371)^{109}$

U.D. = 1

Eg. $\rightarrow$ U.D. of $(106)^{126}$

U.D. = 6

- If U.D. is 4 $\rightarrow (4)^{\text{odd}} \rightarrow \text{U.D.} = 4$
 $(4)^{\text{even}} \rightarrow \text{U.D.} = 6$
- If U.D. is 9 $\rightarrow (9)^{\text{odd}} \rightarrow \text{U.D.} = 9$
 $(9)^{\text{even}} \rightarrow \text{U.D.} = 1$

Eg. $\rightarrow (324)^{171}$
 $(4)^{171} \rightarrow 4 \text{ (U.D.)}$

Eg. $\rightarrow (1019)^{234}$
 $(9)^{234} \rightarrow 1 \text{ (U.D.)}$

- If U.D. is 2, 3, 7, 8 $\rightarrow$ U.D. repeat after every power 4
 $\therefore$ cyclicity $\rightarrow 4$

Remainder	1	2	3	0
Keep power	1	2	3	4

Eg. $\rightarrow (132)^{25}$
 $\frac{25}{4} \rightarrow \text{Remainder} = 1$

$(132)^1 = 2 \text{ (U.D.)}$

Eg. $\rightarrow (333)^{337}$
 $\frac{337}{4} \rightarrow 1 \rightarrow 1$

$(333)^1 \rightarrow 3 \text{ (U.D.)}$

So, $\boxed{\frac{\text{Power}}{4} = \text{Remainder}} \text{ Check}$

Type-4:- Factorial

If $n \geq 15$ always U.D. = 0

Eg. $\rightarrow 124$
U.D. = 0

योग (Summation)

$$1. 1+2+3+4+5+\dots+n = \frac{n(n+1)}{2}$$

$$2. 2+4+6+8+\dots+2n = n(n+1)$$

$$3. 1+3+5+7+\dots+(2n-1) = n^2$$

$$4. 1^2+2^2+3^2+4^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$5. 1^3+2^3+3^3+4^3+\dots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$6. 1^2+3^2+5^2+7^2+\dots+n^2 = \frac{n(n+1)(n+2)}{6}$$

Progression (श्रेणी)

• Arithmetic Progression (A.P.) $\rightarrow a, a+d, a+2d, \dots, a+(n-1)d$

(समान्तर श्रेणी) (n^{th} term) $T_n = a + (n-1)d$

$$(sum) \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [a + l]$$

first last no.

• Geometric Progression (G.P.) $\rightarrow a, ar, ar^2, ar^3, \dots, ar^{(n-1)}$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{if } r > 1$$

$$= \frac{a(1 - r^n)}{1 - r} \quad \text{if } r < 1$$

$$S_\infty = \frac{a}{1 - r}$$

• Harmonic Progression (H.P.) $\rightarrow \frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots, \frac{1}{a+(n-1)d}$

$$T_n = \frac{1}{a+(n-1)d}$$

Mean (माध्य)

Arithmetic mean (A.M.) :- $a, b, c \in A.P.$
(समान्तर माध्य)

$$b = \frac{a+c}{2}$$

Geometric mean (G.M.) :- $a, b, c \in G.P.$
(गुणोत्तर माध्य)

$$b = \sqrt{ac}$$

Harmonic mean (H.M.) :- $a, b, c \in H.P.$
(हारात्मक माध्य)

$$b = \frac{2ac}{a+c}$$

Order (क्रम)

Type-1 :- $\sqrt[3]{3}, \sqrt[4]{4}, \sqrt[6]{6}, \sqrt[12]{12}$, write in descending order.

LCM of $(3, 4, 6, 12) \rightarrow$ out of root $= 12$

$$(3)^{\frac{12}{3}}, (4)^{\frac{12}{4}}, (6)^{\frac{12}{6}}, (12)^{\frac{12}{12}}$$

$$3^4, 4^3, 6^2, 12^1$$

$$81, 64, 36, 12$$

$$81 > 64 > 36 > 12$$

$$\sqrt[3]{3}, \sqrt[4]{4}, \sqrt[6]{6}, \sqrt[12]{12}$$

Type-2 :- $2^{350}, 5^{200}, 3^{300}, 4^{250}$

$$(2^7)^{50}, (5^4)^{50}, (3^6)^{50}, (4^5)^{50}$$

$$(128)^{50}, (625)^{50}, (729)^{50}, (1024)^{50}$$

$$128^{50} < 625^{50} < 729^{50} < 1024^{50}$$

Type-3:- $\sqrt{8} + \sqrt{5}$, $\sqrt{6} + \sqrt{7}$ which is maximum

If sum is equal and sign is +ve

Multiply both numbers

$$8 \times 5 = 40 \quad 6 \times 7 = 42 \quad | \quad 40 < 42$$

Greater will be greater

$$(\sqrt{8} + \sqrt{5}) < (\sqrt{6} + \sqrt{7})$$

Type-4:- $\sqrt{8} - \sqrt{5}$, $\sqrt{7} - \sqrt{6}$ which is maximum

If sum is equal and sign is -ve

Multiply both numbers

$$8 \times 5 = 40 \quad 7 \times 6 = 42$$

Smaller will be greater

$$40 > 42$$

$$(\sqrt{8} - \sqrt{5}) > (\sqrt{7} - \sqrt{6})$$

Non-terminating repeating decimal

$$\bullet \quad 0.7777 \dots = 0.\overline{7} \rightarrow \frac{7}{9} \quad [\text{No. of bar} = \text{No. of 9}]$$

$$0.838383 \dots = 0.\overline{83} \rightarrow \frac{83}{99}$$

$$0.512512512 \dots = 0.\overline{512} \rightarrow \frac{512}{999}$$

$$\bullet \quad 0.86666 \dots = 0.8\overline{6} \rightarrow \frac{86-8}{90} = \frac{78}{90} = \frac{13}{15}$$

$$0.5313131\ldots = 0.5\overline{31} \rightarrow \frac{531-5}{990} = \frac{526}{990}$$

$$0.437777\ldots = 0.43\overline{7} \rightarrow \frac{437-43}{900} = \frac{394}{900}$$

$$7.581\overline{6} \rightarrow 7 + 0.581\overline{6} = 7 + \frac{5816-581}{9000} = 7\frac{5235}{9000}$$

$$11.43\overline{25} \rightarrow 11 + 0.43\overline{25} = 11 + \frac{4325-43}{9900} = 11\frac{4282}{9900}$$

BODMAS Rule

B $\rightarrow$ Brackets

O $\rightarrow$ of

D $\rightarrow$ Division

M $\rightarrow$ Multiplication

A $\rightarrow$ Addition

S $\rightarrow$ Subtraction

Brackets :-

Small $\rightarrow ()$

Middle $\rightarrow \{ \}$

Larger $\rightarrow []$

of means multiplication

Surds & Indices

- $a \times a \times a \dots n \text{ times} = a^n$
- $a^m \times a^n = a^{(m+n)} \quad (a \neq 0)$
- $\frac{a^m}{a^n} = a^{(m-n)} \quad (m > n)$
 $= \frac{1}{a^{(n-m)}} \quad (n > m)$
 $= 1 \quad (m = n)$
- $(a^m)^n = a^{(m \times n)} = a^{(n \times m)} = (a^n)^m$
- $(abc)^n = a^n \times b^n \times c^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$
- $(a^m)^n \neq a^{m^n}$
- $a^{\frac{p}{q}} = a^{\frac{1}{q} \times p} = \left(a^{\frac{1}{q}}\right)^p = (a^p)^{\frac{1}{q}}$
- If $a^m = a^n$, then $m = n$
If $a^m = b^m$, then $a = b$
- $a^0 = 1$
- $a^{-1} = \frac{1}{a} \quad (a \neq 0)$
- $a^{-n} = \frac{1}{a^n}$ & $a^n = \frac{1}{a^{-n}}$
- $\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$
- $(-1)^n = +1 \quad (n = \text{even})$
 $= -1 \quad (n = \text{odd})$
- $\sqrt[n]{a} = a^{\frac{1}{n}}$
- $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b} = a^{\frac{1}{n}} \times b^{\frac{1}{n}} = (ab)^{\frac{1}{n}}$

- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$
- $(\sqrt[n]{a})^m = a^{\frac{m}{n}} = \sqrt[n]{a^m}$
- $(\sqrt[n]{a})^n = a^{\frac{n}{n}} = a$
- $\sqrt[n]{\sqrt[m]{a}} = \sqrt[n]{a^{\frac{1}{m}}} = a^{\frac{1}{mn}}$
- $\left(x\sqrt{\left(y\sqrt{\left(x\sqrt{a}\right)^m}\right)^n}\right)^p = a^{\frac{mnp}{xyz}}$

Componendo & dividendo

- $\frac{a}{b} = \frac{c}{d}$

Apply C & D

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Apply again

$$\frac{a+b+a-b}{a+b-(a-b)} = \frac{c+d+c-d}{c+d-(c-d)}$$

$$\frac{2a}{2b} = \frac{2c}{2d}$$

$$\frac{a}{b} = \frac{c}{d}$$

- $\frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} + \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}} = \frac{2(x+y)}{(x-y)}$

- $\frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} - \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}} = \frac{4\sqrt{xy}}{x-y}$

- $(\sqrt{x} + \sqrt{y})^2 - (\sqrt{x} - \sqrt{y})^2 = 4\sqrt{xy}$

Simplification

$$\bullet \quad \sqrt[n]{\sqrt[n]{\sqrt[n]{\dots \infty}}} = n$$

$$\bullet \quad \left[\sqrt[12]{\left(\sqrt[6]{\left(\sqrt[3]{5^4} \right)^8} \right)^9} \right]^{18} = 5^{\frac{4 \times 8 \times 9 \times 18}{12 \times 6 \times 3}} = 5^{24}$$

$$\bullet \quad 4 + 44 + 444 + \dots \quad \text{Sum of 100 term}$$

$$\boxed{\frac{a}{9} \left[\frac{10(10^n - 1)}{9} - n \right]} \quad \text{Here, } a=4, n=100$$

$$= \frac{4}{9} \left[\frac{10(10^{100} - 1)}{9} - 100 \right]$$

$$\bullet \quad \frac{1}{\underset{5 \wedge 6}{30}} + \frac{1}{\underset{6 \wedge 7}{42}} + \frac{1}{\underset{7 \wedge 8}{56}} + \frac{1}{\underset{8 \wedge 9}{72}} + \frac{1}{\underset{9 \wedge 10}{90}} + \frac{1}{\underset{10 \wedge 11}{110}}$$

No. of terms
(First $\times$ last) factor

difference should be same

$$\Rightarrow \frac{6}{[5 \times 11]} = \frac{6}{55}$$

OR

$$\frac{1}{\text{common diff.}} \left[\frac{1}{\text{1st no. of denominator}} - \frac{1}{\text{last no. of denominator}} \right]$$

$$\bullet \quad \frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \frac{1}{10 \times 13} + \frac{1}{13 \times 16}$$

$$\Rightarrow \frac{1}{3} \left[\frac{1}{1} - \frac{1}{16} \right] \Rightarrow \frac{1}{3} \times \frac{15}{16} \Rightarrow \frac{5}{16}$$

- If denominator is same as multiplier

$$999 \frac{991}{999} \times 999 = ?$$

Step 1 $\rightarrow$ By which multiplied = 999

Step 2 $\rightarrow$ There are as many 0 as there are 9's = 000

Step 3 $\rightarrow$ Difference of 999 - 991 = 8

$$\therefore 999000 - 8 = 998992$$

$$\bullet \frac{1}{1 \times 4 \times 7} + \frac{1}{4 \times 7 \times 10} + \frac{1}{7 \times 10 \times 13} + \frac{1}{10 \times 13 \times 16} + \frac{1}{13 \times 16 \times 19}$$

$$\frac{1}{\text{difference of 1st \& 3rd No. in denominator}} \left[\frac{1}{\text{1st 2no.}} - \frac{1}{\text{last 2no.}} \right]$$

$$\Rightarrow \frac{1}{7-1} \left[\frac{1}{4} - \frac{1}{16} \right]$$

$$\Rightarrow \frac{1}{6} \left[\frac{4-1}{16} \right] \Rightarrow \frac{1}{2} \times \frac{1}{16} \Rightarrow \frac{1}{32}$$

* Squares:-

- If a number ends with 2, 3, 7, 8, it can't be a perfect square.

* Square Mirrors:-

$$14^2 + 87^2 = 78^2 + 41^2$$

$$15^2 + 75^2 = 57^2 + 51^2$$

$$17^2 + 84^2 = 48^2 + 71^2$$

$$26^2 + 97^2 = 79^2 + 62^2$$

$$27^2 + 96^2 = 69^2 + 72^2$$

LCM & HCF

LCM:- Least Common Multiple

3 $\rightarrow$ 3, 6, 9, 12, 15,

4 $\rightarrow$ 4, 8, 12, 16, 20,

$$\text{LCM} = 20$$

- The number when divided by a, b, c leaves remainder ' x ' in each case.

$$= \text{LCM}(a, b, c) \times K + x$$

- LCM of fraction = $\frac{\text{LCM of Numerator (अंश)}}{\text{HCF of Denominator (हर)}}$
(भिन्न)

HCF:- Highest common factor

48 $\rightarrow$ 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

64 $\rightarrow$ 1, 2, 4, 8, 16, 32, 64

$$\text{HCF} = 16$$

HCF by division method:-

HCF of (180, 108) = ?

$$\text{HCF} = 36$$

$$\begin{array}{r} 108 \overline{) 180} \text{ L} \\ \underline{108} \\ 72 \overline{) 108} \text{ L} \\ \underline{72} \\ 36 \overline{) 72} \text{ L} \\ \underline{72} \\ 0 \end{array}$$

- HCF of fraction = $\frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$

- If A & B are two numbers then

$$A \times B = \text{HCF}(A, B) \times \text{LCM}(A, B)$$

Note:-

- HCF of two or more numbers is the greatest number which divide all of them without any remainder.
- LCM of two or more numbers is the smallest number which is divisible by all the given numbers.
- If number a , divides another number b exactly, we say that a is a factor of b . In this case, b is called a multiple of a .
- Two numbers are said to be co-primes if their HCF is 1.
- HCF of a given number always divides its LCM.
- To find the Greatest number that will divide a, b and c leaving remainders x, y & z respectively.

$$\text{Required number} = \text{HCF}(a-x)(b-y)(c-z)$$

- To find the Greatest number that will exactly divide a, b, c .

$$\text{Required number} = \text{HCF}(a, b, c)$$

To find the least number which when divided by a, b, c leaves the remainder x, y and z respectively.

$$\text{Required number} = (\text{LCM of } a, b, c) - k$$

- To find the greatest number that will divide a, b, c leaving the same remainder r in each case.

$$\text{Required number} = \text{HCF}(a-r)(b-r)(c-r)$$

- To find the least number which when divided by a, b, c leaves the same remainder r in each case.

$$\text{Required number} = \text{LCM}(x, y, z) + r$$

- $\text{HCF}[a^n \pm 1, a^m \pm 1] = a^{\text{HCF}(n, m)} \pm 1$

Percentage (प्रतिशत)

$$x\% = \frac{x}{100}, \quad \text{Ex:- } 30\% = \frac{30}{100} = \frac{3}{10}$$

$$\text{Ex:- } 20\% = \frac{20}{100} = \frac{1}{5}$$

- $\text{Percentage} = \frac{\text{Value}}{\text{Total Value}} \times 100$
- $\% \text{ Increase} = \frac{\text{Increase}}{\text{Initial Value}} \times 100$
- $\% \text{ Decrease} = \frac{\text{Decrease}}{\text{Initial Value}} \times 100$
- $\% \text{ Change} = \frac{\text{Change}}{\text{Initial Value}} \times 100$
- If two values are respectively $x\%$ and $y\%$ more than a third value, then the first is $\left[\frac{(100+x)}{(100+y)} \times 100 \right]\%$ of the second.
- If we change a number by $x\%$ and $y\%$ respectively, Then net $\% \text{ change} \rightarrow \left[x + y + \frac{xy}{100} \right]$
[If increase $\rightarrow +ve$ sign
If decrease $\rightarrow -ve$ sign]
- Net $\% \text{ change of } x\%, y\% \text{ and } z\% \text{ is}$
 $\left[(x+y+z) + \frac{xy+yz+zx}{100} + \frac{xyz}{10000} \right]\%$
- If an amount is increased by $x\%$ and then it is decreased by $x\%$ again, then percentage change will be a decrease of $\left[\frac{x^2}{10} \right]\%$.

- If the population/cost of a certain town/article, is P and the annual increment rate is $x\%$, then

(i) After 't' years population/cost = $P \left[1 + \frac{x}{100} \right]^t$

(ii) Before 't' years population/cost = $\frac{P}{\left[1 + \frac{x}{100} \right]^t}$

- If the population of a town/cost of a article is P and it decreases/reduces at the rate of $x\%$ annually, then,

(i) After 't' years population/cost = $P \left[1 - \frac{x}{100} \right]^t$

(ii) Before 't' years population/cost = $\frac{P}{\left[1 - \frac{x}{100} \right]^t}$

- If x is $A\%$ more than y , then y is more than ' x ' by required % = $\left[\frac{A}{(100-A)} \times 100 \right] \%$

- If x is $A\%$ less than y , then y is less than ' x ' by required % = $\left[\frac{A}{(100+A)} \times 100 \right] \%$

- If the length of the rectangle is change by $x\%$ and the breadth of the rectangle is change by $y\%$, then the % change in area :- $\left[\pm x \pm y + \frac{xy}{100} \right] \%$
 $\left[\begin{array}{l} \text{If increase} \rightarrow +ve \text{ sign} \\ \text{If decrease} \rightarrow -ve \text{ sign} \end{array} \right]$

- If the side of triangle, rectangle, square, circle, rhombus are increased by $x\%$, then its area is increased by $\left[2x + \left(\frac{x^2}{100} \right) \right] \%$

- A candidate scoring $x\%$ in an exam fails by 'a' marks, while another candidate who scores $y\%$ marks gets 'b' marks more than the minimum pass marks. Then the maximum marks for the exam are :-

$$\left[\frac{100 \times (a+b)}{y-x} \right]$$

- In an exam $x\%$ students failed in English and $y\%$ students failed in Maths. If $z\%$ of students failed in both, Then the % of passed students in both subjects is = $100 - (x+y-z)$ or $(100-x) + (100-y) + z$

- If the price of a commodity is diminished by $x\%$ and its consumption is increased by $y\%$.

OR
If the price of a commodity is increased by $x\%$ and its consumption is decreased by $y\%$

Then the effect on revenue is

$$\left[\text{Increase \% Value} - \text{Decrease \% Value} - \frac{(\text{Inc \% Value} \times \text{Dec \% Value})}{100} \right]$$

Percentage - Fraction Table

Fraction	Percentage	Fraction	Percentage
1	100%	$\frac{1}{25}$	4%
$\frac{1}{2}$	50%	$\frac{1}{50}$	2%
$\frac{1}{3}$	$33\frac{1}{3}\%$	$\frac{2}{3}$	$66\frac{2}{3}\%$
$\frac{1}{4}$	25%	$\frac{4}{5}$	80%
$\frac{1}{5}$	20%	$\frac{3}{4}$	75%
$\frac{1}{6}$	$16\frac{2}{3}\%$	$\frac{5}{11}$	$45\frac{5}{11}\%$
$\frac{1}{7}$	$14\frac{2}{7}\%$	$\frac{7}{11}$	$63\frac{7}{11}\%$
$\frac{1}{8}$	$12\frac{1}{2}\%$	$\frac{10}{11}$	$90\frac{10}{11}\%$
$\frac{1}{9}$	$11\frac{1}{9}\%$	$\frac{4}{9}$	$44\frac{4}{9}\%$
$\frac{1}{10}$	10%	$\frac{7}{9}$	$77\frac{7}{9}\%$
$\frac{1}{11}$	$9\frac{1}{11}\%$	$\frac{5}{6}$	$83\frac{1}{3}\%$
$\frac{1}{12}$	$8\frac{1}{3}\%$	$\frac{4}{7}$	$57\frac{1}{7}\%$
$\frac{1}{13}$	$7\frac{6}{13}\%$	$\frac{5}{8}$	$62\frac{1}{2}\%$
$\frac{1}{14}$	$7\frac{1}{7}\%$	$\frac{3}{8}$	$37\frac{1}{2}\%$
$\frac{1}{15}$	$6\frac{2}{3}\%$	$\frac{5}{4}$	125%
$\frac{1}{16}$	$6\frac{1}{4}\%$	$\frac{6}{5}$	120%
$\frac{1}{17}$	$5\frac{15}{17}\%$		
$\frac{1}{18}$	$5\frac{5}{9}\%$		
$\frac{1}{19}$	$5\frac{5}{19}\%$		
$\frac{1}{20}$	5%		

Profit & Loss (लाभ-हानि)

- Profit = Selling Price (SP) - Cost Price (CP)

- Loss = Cost Price (CP) - Selling Price (SP)

- Profit % = $\left[\frac{\text{Profit}}{\text{Cost Price}} \times 100 \right] \%$

- Loss % = $\left[\frac{\text{Loss}}{\text{Cost Price}} \times 100 \right] \%$

- If Cost Price = Rs. x , Profit = $R\%$, then
Selling Price = $x \left[\frac{100+R}{100} \right]$

- If Cost Price = Rs. x , Loss = $R\%$, then
Selling Price = $x \left[\frac{100-R}{100} \right]$

- If Selling Price = Rs. x , Profit = $R\%$, then
Cost Price = $\frac{x \times 100}{(100+R)}$

- If Selling Price = Rs. x , Loss = $R\%$, then
Cost Price = $\frac{x \times 100}{(100-R)}$

- Profit % = $\left[\frac{\text{Error}}{\text{True Value} - \text{Error}} \times 100 \right] \%$

- Profit % = $\left[\frac{\text{True weight} - \text{false weight}}{\text{false weight}} \times 100 \right] \%$

If Cost Price of x articles = Selling Price of y articles
then Profit % = $\left[\frac{(x-y)}{y} \times 100 \right] \%$

- If two similar objects one at a loss of $x\%$ and another at a gain of $x\%$, then he always incurs loss in this transaction and

$$\text{Loss \%} = \left[\frac{x^2}{100} \right] \%$$

- If a vendor used to sell his articles at $x\%$ loss on cost price but uses y grams instead of z grams, then
Profit or loss $\% = \left[(100 - x) \frac{y}{z} - 100 \right] \%$

- Discount = Marked Price - Selling Price

$$\text{Discount \%} = \left[\frac{\text{MP} - \text{SP}}{\text{MP}} \times 100 \right] \%$$

- Discount calculated on MP always.
- If shopkeeper does not allow any discount then $\text{MP} = \text{SP}$.
- Relation between CP and MP:-

$$\frac{\text{CP}}{(100 - \text{Discount}\%)} = \frac{\text{MP}}{(100 \pm \text{Profit/Loss}\%)}$$

- If successive discount of $x\%$ and $y\%$, then
Overall discount $\% = \left[x + y - \frac{xy}{100} \right] \%$
- If successive discount of $x\%$, $y\%$ and $z\%$, then
Overall discount $\% = x + y + z - \frac{xy}{100} - \frac{yz}{100} - \frac{zx}{100} + \frac{xyz}{1000}$
- Marked Price = $\frac{\text{Selling Price} \times 100}{(100 - \text{Discount}\%)}$

- Selling Price = $\frac{\text{Marked Price} \times (100 - \text{Discount}\%)}{100}$

- 'y' articles (quantity/number) are given free on purchasing 'x' articles. Then,

$$\text{Discount}\% = \left[\frac{x \times 100}{x + y} \right] \%$$

Ratio & Proportion

- If the ratio of $A:B = 2:3$
then $A = 2x$, $B = 3x$

- If an amount R is to be divided between A and B in the ratio $m:n$ then

$$\text{Part of } A = \frac{m}{m+n} \times R$$

$$\text{Part of } B = \frac{n}{m+n} \times R$$

- If $a:b::c:d$, then a and d are called extremes and b and c are called means.

Product of extremes = Product of means

$$\boxed{ad = bc}$$

- If x is added to all a, b, c, d (numbers) so that these become proportional respectively, then

$$\boxed{\frac{a+x}{b+x} :: \frac{c+x}{d+x}}$$

- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$ then each ratio $= \frac{a+c+e+\dots}{b+d+f+\dots}$

- Let $x:y$ and $P:Q$ are two ratios, then $\boxed{Px:Qy}$ is called mixed ratio.

- Duplicate ratio:- Duplicate ratio of $\boxed{a:b}$ is $a^2:b^2$

- Subduplicate ratio:- The square root of a certain ratio is called its subduplicate.

$$\text{Subduplicate ratio of } \boxed{a:b} = \sqrt{a}:\sqrt{b}$$

- Triplicate Ratio :- The cube of a certain ratio is called triplicate ratio.

The triplicate ratio of $a:b = a^3:b^3$

- Subtriplicate Ratio :- The cube root of a certain ratio is called subtriplicate ratio.

The Subtriplicate ratio of $a:b = \sqrt[3]{a}:\sqrt[3]{b}$

- Inverse Ratio :- The reciprocal of quantities of ratio is called its inverse.

Reciprocal or Inverse ratio of $a:b = \frac{1}{a}:\frac{1}{b}$

- Componendo and Dividendo :-

$$\text{If } \frac{a}{b} = \frac{c}{d}$$

$$\text{then, } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

- Mean Proportion :-

Mean Proportion of a and $b = \sqrt{ab}$

- Third Proportion :-

Third Proportion of a and $b = \frac{b^2}{a}$

- Fourth Proportion :-

Fourth Proportion of a, b and $c = \frac{bc}{a}$

- First Proportion :-

First Proportion of a, b and $c = \frac{ab}{c}$

• Compound Ratio:- Compound ratio of $a:b, c:d, e:f$
 $= \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = \frac{ace}{bdf} \Rightarrow ace:bdf$

• If $\frac{a}{b} = \frac{c}{d}$, then

(i) $ad = bc$

(ii) $\frac{a}{c} = \frac{b}{d}$

(iii) $\frac{a+b}{b} = \frac{c+d}{d}$

(iv) $\frac{a-b}{b} = \frac{c-d}{d}$

(v) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

• Trick:- If $A:B = 2:3, B:C = 4:5$, then $A:B:C$

$$\begin{array}{rcl} A : B : B & = & 2 : 3 : 3 \\ B : B : C & = & \times 4 : 4 : 5 \\ \hline & & 8 : 12 : 15 \end{array}$$

• If $A:B = 1:3, B:C = 4:5, C:D = 2:7$, then $A:B:C:D = ?$

$$\begin{array}{rcl} A : B : B : B & = & 1 : 3 : 3 : 3 \\ B : B : C : C & = & 4 : 4 : 5 : 5 \\ C : C : C : D & = & \times 2 : 2 : 2 : 7 \\ \hline & & 8 : 24 : 30 : 105 \end{array}$$

• Trick:- If $3A = 4B = 5C$, then $A:B:C = ?$

$$\begin{array}{lcl} A : B : C = 4 \times 5 & : & 3 \times 5 & : & 3 \times 4 \\ \text{for A} & & \text{for B} & & \text{for C} \\ \text{multiply the co-} & & \text{multiply the co-} & & \text{multiply the co-} \\ \text{efficient of B \& C} & & \text{efficient of A \& C} & & \text{efficient of A \& B} \\ \Rightarrow 20 : 15 : 12 \end{array}$$

Simple Interest

P = Principal, R = Rate of interest p.a., T = time,
S.I. = Simple Interest, A = Amount

- $S.I. = \frac{P \times R \times T}{100}$
- $A = P + S.I.$
- When interest is payable half-yearly, then rate will be half and time will be twice.
- When interest is payable quarterly, then rate will be one-fourth and time will be four times.
- If there are distinct rates of interest for distinct time periods i.e.
Rate for 1st t_1 year = $R_1\%$, Rate for 2nd t_2 year = $R_2\%$
Rate for 3rd t_3 year = $R_3\%$, Rate for 4th t_4 year = $R_4\%$
then, total S.I. for 4 years = $\frac{P(R_1 t_1 + R_2 t_2 + R_3 t_3 + R_4 t_4)}{100}$
- If a certain sum becomes 'n' times of itself in T years on simple interest, then the rate percent per annum is.

$$R = \frac{(n-1)}{T} \times 100 \text{ and,}$$

$$T = \frac{(n-1)}{R} \times 100$$

- If a sum with simple interest rate, amounts to 'A' in t_1 years and 'B' in t_2 years, then

$$R\% = \frac{(B-A) \times 100}{At_2 - Bt_1} \text{ and}$$

$$P = \frac{At_2 - Bt_1}{t_2 - t_1}$$

- If a sum is to be deposited in equal installments, then Equal Installments = $\frac{A \times 200}{T[200 + (T-1)r]}$
- To find the rate of interest under current deposit plan,

$$r = \frac{S.I. \times 2400}{n(n+1) \times (\text{deposited amount})}$$

Compound Interest

A = Amount, P = Principal, r = rate of C. I.

t = no. of years

- $A = P \left[1 + \frac{r}{100} \right]^t$
- $C.I. = A - P$
- $C.I. = P \left[\left(1 + \frac{r}{100} \right)^t - 1 \right]$
- Compound interest is calculated on four basis:-

	Rate	Time (n)
Annually	$r\%$	t years
Half-yearly	$\frac{r}{2}\%$	$t \times 2$ years
Quarterly	$\frac{r}{4}\%$	$t \times 4$ years
Monthly	$\frac{r}{12}\%$	$t \times 12$ years
- If there are distinct 'rates of interest' for distinct time period i.e.
 - Rate of 1st year $\rightarrow r_1\%$
 - Rate of 2nd year $\rightarrow r_2\%$
 - Rate of 3rd year $\rightarrow r_3\%$then,
$$A = P \left[1 + \frac{r_1}{100} \right] \left[1 + \frac{r_2}{100} \right] \left[1 + \frac{r_3}{100} \right]$$

- If the time is in fractional form:-

$$t = n \frac{a}{b}$$

$$A = P \left[1 + \frac{r}{100} \right]^n \left[1 + \frac{r \times \frac{a}{b}}{100} \right]$$

- A certain sum becomes 'm' times of itself in 't' years on compound interest then, it will take to becomes m^n times of itself in txn years.

Eg. $\rightarrow$ 4 years $\rightarrow$ 2 times

? $\rightarrow$ 8 times

$$8 = 2^3$$

So, in $4 \times 3 = 12$ years, it will become 8 times.

- If a sum becomes 'n' times of itself in 't' years on compound interest, then

$$R\% = \left[n^{\frac{1}{t}} - 1 \right] \times 100$$

- The difference between S.I. and C.I. on a sum 'P' at the rate of R% p.a.,

$$\text{For 2 years, difference (C.I. - S.I.)} = P \left[\frac{R}{100} \right]^2$$

$$\text{For 3 years, C.I. - S.I.} = P \left[\frac{R}{100} \right]^2 \times \left[3 + \frac{R}{100} \right]$$

- A certain sum at C.I. becomes x times in n_1 year and y times in n_2 years, then $x^{\frac{1}{n_1}} = y^{\frac{1}{n_2}}$

- If on compound interest, a sum becomes Rs. A in 'a' years and Rs. B in 'b' years. then,

(i) If $b-a=1$, then $R\% = \left[\frac{B}{A} - 1 \right] \times 100\%$

(ii) If $b-a=2$, then $R\% = \left[\sqrt{\frac{B}{A}} - 1 \right] \times 100\%$

(iii) If $b-a=n$, then $R\% = \left[\left(\frac{B}{A} \right)^{\frac{1}{n}} - 1 \right] \times 100\%$

where n is a whole number.

- If a sum becomes 'n' times of itself in 't' years on compound interest, then

$$R\% = \left[n^{\frac{1}{t}} - 1 \right] \times 100\%$$

- Installment:-

for 2 years, Each ^{annual} installment =
$$\frac{P}{\left(\frac{100}{100+r} \right) + \left(\frac{100}{100+r} \right)^2}$$

for 3 years, Each annual installment

$$= \frac{P}{\left(\frac{100}{100+r} \right) + \left(\frac{100}{100+r} \right)^2 + \left(\frac{100}{100+r} \right)^3}$$

Average

- $$\text{Average} = \frac{\text{Sum of all observation}}{\text{Total No. of all observation}}$$

- If the average of ' n_1 ' numbers is a_1 , and the average of ' n_2 ' numbers is a_2 , then average of total numbers n_1 and n_2 is

$$\text{average} = \frac{n_1 a_1 + n_2 a_2}{n_1 + n_2}$$

- No. of data $\rightarrow n_1 \quad n_2 \quad n_3 \quad n_4 \quad \dots$
Average $\rightarrow a_1 \quad a_2 \quad a_3 \quad a_4 \quad \dots$

$$\text{Net Average} = \frac{n_1 a_1 + n_2 a_2 + n_3 a_3 + n_4 a_4 + \dots}{n_1 + n_2 + n_3 + n_4 + \dots}$$

- The average of n consecutive natural numbers from 1 Or Average of $1, 2, 3, \dots, n = \frac{n+1}{2}$

- The average of square of n consecutive natural numbers from 1 Or Average of $1^2, 2^2, 3^2, \dots, n^2$
$$= \frac{(n+1)(2n+1)}{6}$$

- The average of cubes of first n consecutive natural numbers from 1 Or Average of $1^3, 2^3, 3^3, \dots, n^3$
$$= \frac{n(n+1)^2}{4}$$

- The average of first n consecutive even numbers Or average of $2, 4, 6, \dots, 2n = (n+1)$

- The average of first 'n' consecutive odd natural numbers Or Average of $1, 3, 5, \dots, (2n-1) = n$
- If the difference of the numbers is same,

$$\text{Average} = \frac{\text{First} + \text{Last}}{2}$$

[Arrange the numbers in ascending order]
- Average of 1^{st} 'n' multiples of certain numbers x

$$= \frac{x(1+n)}{2}$$
- Average of square of 1^{st} n even number

$$= \frac{2n(n+1)(2n+1)}{3}$$
- Average of cube of 1^{st} n even number $= 2n(n+1)^2$
- Average of square of 1^{st} n odd number -

$$= \frac{n(n+1)(2n-1)}{3}$$
- Average of cube of 1^{st} n odd number $= n(2n^2-1)$
- Average of 1 to n odd number $= \frac{\text{Last odd no.} + 1}{2}$
- Average of 1 to n even number $= \frac{\text{Last even no.} + 2}{2}$
- Bowling Average $= \frac{\text{Total runs given}}{\text{Total Wicket taken}}$
- Batting Average $= \frac{\text{Total runs scored}}{\text{Total number of inning played}}$

- If a person goes from P to Q with speed of x km/h and returns from Q to P with speed of y km/h, then average speed = $\frac{2xy}{x+y}$
- If a distance is travelled with three different speeds a km/h, b km/h and c km/h, then average speed of total journey = $\frac{3abc}{ab+bc+ca}$ km/h
- If average of n observations is a but the average becomes b when one observation is eliminated, then value of eliminator observation = $n(a-b)+b$
- If average of n observations is a but the average becomes b when a new observation is added then value of added observation = $n(b-a)+a$
- If average of n numbers is m but later on it was found that a number ' a ' was misread as ' b '. The correct average = $m + \frac{(a-b)}{n}$

Time & Work

- If A completes a piece of work in x days and B completes the same work in y days. then,

Total time taken to complete the work by A and B both = $\left[\frac{xy}{x+y} \right]$

- If A can do a work in x days, B can do the same work in y days, C can do the same work in z days then, total time taken by A, B and C to complete the work together = $\frac{xyz}{xy+yz+zx}$

- If A can finish $\frac{m}{n}$ part of the work in D days. Then, total time taken to finish the work by A = $\left[\frac{n}{m} \times D \right]$ days

- If A and B can do a work in x days, B and C can do the same work in y days, C and A can do the same work in z days. Then total time taken by A, B and C to finish the work together = $\frac{2xyz}{xy+yz+zx}$

- If A alone can do a certain work in x days and A and B together can do the same work in y days, then time taken by B alone to finish the work = $\left[\frac{xy}{x-y} \right]$ days

- Efficiency $\propto \frac{1}{\text{time}}$

Efficiency Ratio of x and $y = a:b$

Time taken by x and y to finish the work $= \frac{1}{x} : \frac{1}{y}$

- If A can do a work in x days and B is $R\%$ more efficient than A then time taken by B alone to complete the work $= x \left[\frac{100}{100+R} \right]$

- A can do a work in m days and B can do the same work in n days. If they work together and total wages is R , then

$$\text{Part of } A = \frac{n}{(m+n)} \times R$$

$$\text{Part of } B = \frac{m}{(m+n)} \times R$$

- If A , B and C finish the work in m , n and p days respectively and they receive the total wages R , then the ratio of their wages $= \frac{1}{m} : \frac{1}{n} : \frac{1}{p}$

- $A + B = x$ days

$$A \rightarrow (x+a) \text{ days}$$

$$B \rightarrow (x+b) \text{ days}$$

$$\text{then } x = \sqrt{ab}$$

- If M_1 men finish W_1 work in D_1 days working T_1 time each day and M_2 men finish W_2 work in D_2 days, working T_2 time each day then

$$\frac{M_1 D_1 T_1}{W_1} = \frac{M_2 D_2 T_2}{W_2}$$

- If A men or B boys or C women can do a certain work in x days then A_1 men B_1 boys and C_1 women can do the same work in

$$\Rightarrow \left[\frac{x}{\frac{A_1}{A} + \frac{B_1}{B} + \frac{C_1}{C}} \right] \text{ days}$$

Pipe and Cistern

- Two taps A and B can fill a tank in x hours and y hours respectively. If both the taps are opened together, then how much time it will take to fill the tank?

$$\text{Required time} = \left[\frac{xy}{(x+y)} \right] \text{ hours}$$

- Two taps A and B can empty a tank in x hours and y hours respectively. If both the taps are opened together, then time taken to empty the tank = $\left[\frac{xy}{(x+y)} \right]$ hours.

- If a pipe fills a tank in ' x ' hours but it takes ' t ' more hours to fill it due to leakage in tank. If tank is filled completely, the time taken to empty the tank = $\frac{x(x+t)}{t}$ hours

- Tap A can fill a tank in x hours and tap B can empty the tank in y hours. Time taken to fill the tank, when both are opened

(a) If $x > y$

$$= \left[\frac{xy}{x-y} \right]$$

(b) If $x < y$ [In this case tank will be empty]

$$= \left[\frac{xy}{y-x} \right]$$

If pipe A fill the tank in 'T' hours and rate of fill/empty the tank is 'R' litre/hour. then,

$$\text{Capacity of tank} = T \times R$$

Time, Speed and Distance

- Distance = Speed $\times$ time
- Time = $\frac{\text{Distance}}{\text{Speed}}$
- Speed = $\frac{\text{Distance}}{\text{time}}$
- $1 \text{ Km/h} = \frac{5}{18} \text{ m/sec.}$
- $1 \text{ m/sec} = \frac{18}{5} \text{ Km/h}$
- If $D = \text{constant}$, then
 $S \propto \frac{1}{T}$, $T \propto \frac{1}{S}$
- If $T = \text{constant}$, then
 $D \propto S$
- If $S = \text{constant}$, then
 $D \propto T$
- Average Speed = $\frac{\text{Total Distance}}{\text{Total Time}}$
- If a person covers two equal distance with $x \text{ Km/h}$ and $y \text{ Km/h}$, then
Average Speed = $\frac{2xy}{x+y}$

- If a person covers three equal distances with x , y and z speed, then

$$\text{Average Speed} = \frac{3xyz}{xy + yz + zx}$$

- If A and B starts walking towards each other. After meeting each other. A covered his remaining distance in t_1 time and B covered his remaining distance in t_2 time, then ratio of their speed $\frac{A}{B} = \sqrt{\frac{t_2}{t_1}}$

If they meet after ' t ' hours then $t = \sqrt{t_2 \times t_1}$

- If an object increases/decreases its speed from x km/h to y km/h to cover a distance in t_2 hours in place of t_1 hours, then [Here $(t_2 - t_1)$ will be given].

$$\text{Distance} = \frac{xy}{\text{Difference of } x \text{ and } y} \times (\text{change in time})$$

- If an object travels certain distance with the speed of $\frac{A}{B}$ of its original speed and reaches its destination t hours before or after, then the time taken by object travelling at original speed is

$$\text{Time} = \frac{A}{\text{Difference of } A \text{ and } B} \times \text{time (in hours)}$$

- If a man travels at the speed of ' s_1 ', he reaches his destination t_1 late while he reaches t_2 before when he travels at ' s_2 ' speed, then the distance between the two places is

$$\text{Distance} = \frac{(s_1 \times s_2) \times (t_1 + t_2)}{(s_2 - s_1)}$$

- When the two objects travel in same direction
Relative speed = difference of speeds
- When the two objects travel in opposite direction
Relative speed = sum of both speeds
- Formula to calculate the no. of rounds.
Circular distance = Circumference $\times$ No. of rounds
 $D = 2\pi r \times n$
- If any one overtakes or follows another, then
time taken to catch = $\frac{\text{Distance between them}}{\text{Relative speed}}$
meeting time = $\frac{(\text{Speed of 1st traveller}) \times \text{time}}{\text{Difference of speeds}}$
Total travelled distance to catch the thief
= $\frac{\text{Product of speeds} \times \text{time}}{\text{Difference of speeds}}$
- Distance between poles = $(n-1) \times$
 n = No. of poles, $\times$ = distance b/w consecutive two poles

Problem based on Train

- When train passes a pole or stationary man,
Crossing time = $\frac{\text{length of train}}{\text{speed of train}}$

- When train passes a bridge / platform

$$\text{Crossing time} = \frac{\text{Length of train} + \text{Length of bridge / platform}}{\text{Speed of train}}$$

$$\text{Distance covered} = \text{Length of train} + \text{length of bridge / platform}$$

- When a train passes another train in opposite direction, time to cross each other

$$T = \frac{L_1 + L_2}{S_1 + S_2}$$

$$\text{Distance covered} = L_1 + L_2$$

- When a train passes another train in same direction, time taken to cross each other

$$T = \frac{L_1 + L_2}{S_1 - S_2}$$

- When a train passes a person sitting in another moving train

$$T = \frac{L_1}{\text{Relative Speed}}$$

- If a train crosses a standing man/a pole in t_1 sec time and crosses P m long platform in t_2 sec time, then the length of the train

$$= \frac{P \times t_1}{t_2 - t_1}$$

- If two trains of same lengths are coming from same direction and cross a man in t_1 and t_2 seconds, then time taken by both the trains to cross each other

$$= \frac{2 \times \text{Product of time}}{\text{Difference of time}}$$

- If two trains of same length are coming from opposite directions and cross a man in t_1 seconds and t_2 seconds then time taken by both trains to cross each other

$$= \frac{2 \times \text{Product of time}}{\text{Sum of time}}$$

Excluding stoppage, the average speed of a train is u and with stoppage its average speed is v .

Then, the stoppage time per hour

$$= \frac{\text{Difference between their average speed}}{\text{Speed without stoppage}}$$

$$= \frac{u - v}{u} \quad [\text{where, } u > v \text{ and } u, v \neq 0]$$

Boat and Stream

- Speed of boat in still water = x km/h
Speed of stream = y km/h
Speed of boat in downstream = $(x+y)$ km/h
Speed of boat in upstream = $(x-y)$ km/h

- If speed of boat in downstream = a km/h
Speed of boat in upstream = b km/h

then,

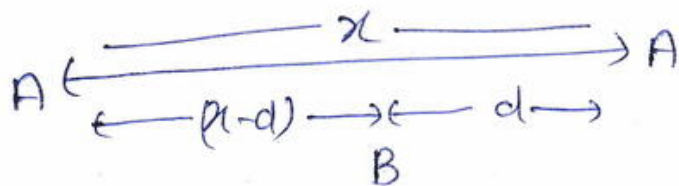
$$\text{Speed of boat in still water} = \frac{a+b}{2} \text{ km/h}$$

$$\text{Speed of stream} = \frac{a-b}{2} \text{ km/h}$$

- If the speed of a boat or swimmer in still water is a km/h and the speed of stream is b km/h. Then average speed in going to a certain place and coming back to starting point is given by = $\frac{(a+b)(a-b)}{a}$ km/h

Race

- In a race of 'x' m, A beats B by 'd' m.



Time is same.

Hence, $D \propto S$

So, ratio of distance & speed will be same.

$$\frac{D_A}{D_B} = \frac{S_A}{S_B} = \frac{x}{x-d}$$

Eg. $\rightarrow$ 400 m, A give a start of 50 m to B and still beat him by 80 m.

$$\frac{D_A}{D_B} = \frac{400}{(400-80-50)} = \frac{400}{270} = \frac{40}{27}$$

$$\frac{S_A}{S_B} = \frac{40}{27}$$

- If in a race of length L , the time taken by A and B be t_A and t_B ($t_B > t_A$), then the distance by which A beats B given by,

$$d = \left[\frac{L}{t_B} \right] (t_B - t_A)$$

$$\text{Or, } d = B's \text{ speed} \times (t_B - t_A)$$

- If in a race of length L , A can give B a start of 'b' and C a start of 'c' then the start that B can give C.

$$= L \left[\frac{c-b}{L-b} \right]$$

- If A gives B a start of distance 'd' and still beats him by time 't' in a race of length 'L', then B's speed is

$$S_B = \frac{L-d}{\frac{L}{S_A} + t} = \frac{\text{Distance covered by B}}{\text{Total time taken by B}}$$

where, $S_A = A$'s speed

- A and B walk around a circle of circumference 'p' with speeds S_A and S_B respectively. If they start simultaneously from the same point, the time after which they will be together again for the first time

$$= \frac{p}{S_A - S_B} = \frac{\text{Circumference}}{\text{Relative Speed}}$$

Partnership

- Profit $\propto$ Investment / Capital
- Profit $\propto$ Time
- Profit $\propto$ Capital $\times$ Time

So, Profit = Capital $\times$ Time

- Profit Ratio = $M_1 T_1 : M_2 T_2 : M_3 T_3 : M_4 T_4$

$M_1, M_2, M_3, M_4 \rightarrow$ Investments by different person

$T_1, T_2, T_3, T_4 \rightarrow$ Time

Mixture & Alligation

- Ratio of milk and water = 2:3

$$\text{concentration of milk} = \frac{\text{milk}}{\text{Total}} \times 100$$

$$= \frac{2}{(2+3)} \times 100 = \frac{2}{5} \times 100$$

$$= 40\%$$

$$\text{concentration of water} = \frac{\text{Water}}{\text{Total}} \times 100$$

$$= \frac{3}{5} \times 100 = 60\%$$

- Let a vessel contains x litres of pure liquid. From this 'y' litre taken out and replaced by an equal amount of water and this process is repeated n times, then

$$\text{quantity of pure liquid left} = x \left[\frac{x-y}{x} \right]^n$$

- If taken out y_1 , y_2 and y_3 litres and replaced by an equal amount of water, then

$$\text{final quantity of liquid} = x \left(\frac{x-y_1}{x} \right) \left(\frac{x-y_2}{x} \right) \left(\frac{x-y_3}{x} \right) \dots$$

- If x is initial amount of liquid. p is the amount which is drawn, and this process is repeated n times such that the resultant mixture is in the ratio $a:b$, then

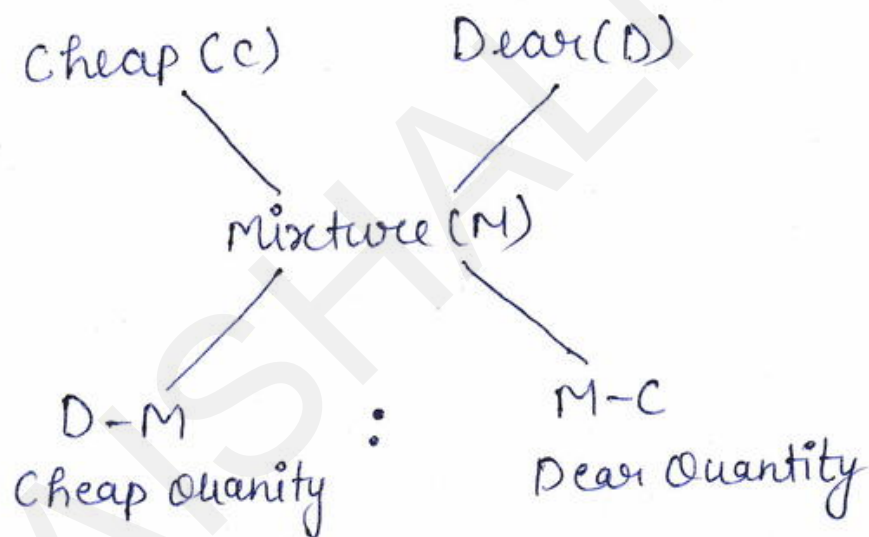
$$\frac{a}{a+b} = \left(\frac{x-p}{x} \right)^n$$

- The amount of acid/milk is $x\%$ in 'M' litre mixture. How much water should be mixed in it so that percentage amount of acid/milk would be $y\%$?

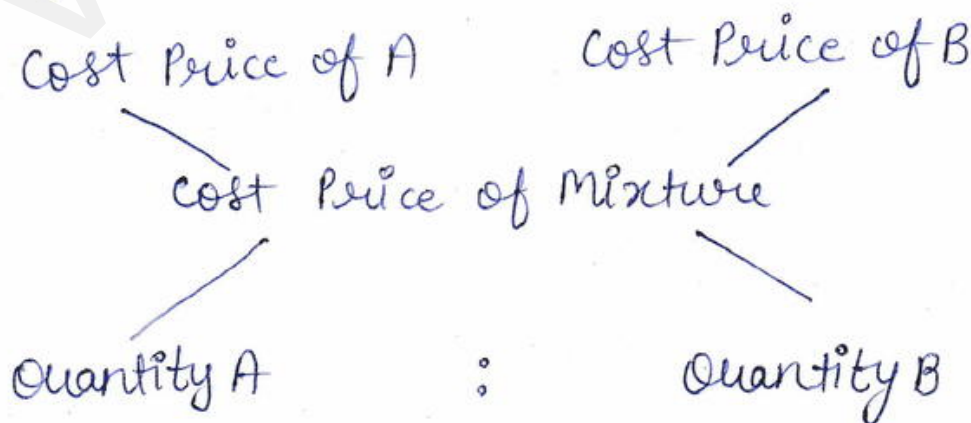
$$\text{Amount of water} = \frac{M(x-y)}{y}$$

- The cost of cheap object is Rs. C/kg and the cost of dear object is Rs. D/kg . If the mixture of both object costs Rs. M/kg , then

$$\frac{\text{Cheap object}}{\text{Dear object}} = \frac{D-M}{M-C}$$



- Alligation in Profit/Loss



Algebra

- $(a+b)^2 = a^2 + b^2 + 2ab$
- $(a+b)^2 = (a-b)^2 + 4ab$
- $(a-b)^2 = a^2 + b^2 - 2ab$
- $(a-b)^2 = (a+b)^2 - 4ab$
- $a^2 - b^2 = (a+b)(a-b)$
- $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
- $(a+b)^2 - (a-b)^2 = 4ab$
- $a^2 + b^2 = (a+b)^2 - 2ab = (a-b)^2 + 2ab$
- $(ax+by)^2 + (ay-bx)^2 = (a^2+b^2)(x^2+y^2)$
- $\frac{a+b}{a-b} + \frac{a-b}{a+b} = \frac{2(a^2+b^2)}{(a^2-b^2)}$
- $\frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{4ab}{(a^2-b^2)}$
- $a^4 + b^4 = (a^2+b^2)^2 - 2a^2b^2 = (a^2-b^2)^2 + 2a^2b^2$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$
- $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
- $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$
- $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$

- $a+b = \frac{a^3+b^3}{a^2+b^2-ab}$

- $a-b = \frac{a^3-b^3}{a^2+b^2+ab}$

- $(a+b+c)^3 = a^3+b^3+c^3 + 3(a+b)(b+c)(c+a)$

- If $x^2+y^2+z^2=0$, then, $x=0, y=0, z=0$

- $a^2+b^2+c^2-ab-bc-ca = \frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$

- If $a^2+b^2+c^2-ab-bc-ca=0$
OR $a^2+b^2+c^2=ab+bc+ca$
then, $a=b=c$

- $a^3+b^3+c^3-3abc = (a+b+c)[a^2+b^2+c^2-(ab+bc+ca)]$
 $= (a+b+c)[(a+b+c)^2-3(ab+bc+ca)]$
 $= \frac{a+b+c}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$

- If $a^3+b^3+c^3-3abc=0$
OR $a^3+b^3+c^3=3abc$

then,

$$\begin{array}{c} \swarrow \quad \searrow \\ a+b+c=0 \quad a=b=c \end{array}$$

- $(a^4+a^2b^2+b^4) = (a^2+ab+b^2)(a^2-ab+b^2)$

- If $(x-a)^2+(y-b)^2+(z-c)^2=0$
then, $x=a, y=b, z=c$

Interchange formula

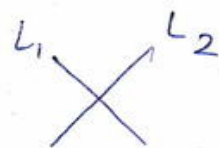
- If $x + \frac{1}{x} = N$, $x^2 + \frac{1}{x^2} = N^2 - 2$
- If $x - \frac{1}{x} = N$, $x^2 + \frac{1}{x^2} = N^2 + 2$
- If $x + \frac{1}{x} = N$, $x^3 + \frac{1}{x^3} = N^3 - 3N$
- If $x - \frac{1}{x} = N$, $x^3 - \frac{1}{x^3} = N^3 + 3N$
- If $x + \frac{1}{x} = N$, $x - \frac{1}{x} = \pm \sqrt{N^2 - 4}$
- If $x - \frac{1}{x} = N$, $x + \frac{1}{x} = \pm \sqrt{N^2 + 4}$
- If $x + \frac{1}{x} = N$, $x^5 + \frac{1}{x^5} = N^5 - 5N^3 + 5N$
- If $x - \frac{1}{x} = N$, $x^5 - \frac{1}{x^5} = N^5 + 5N^3 + 5N$
- If $x + \frac{1}{x} = 2$, then $x = 1$ (always)
- If $x + \frac{1}{x} = -2$, then $x = -1$ (always)
- If $x + \frac{1}{x} = 1$, then $x^3 = -1$ (always)
- If $x + \frac{1}{x} = -1$, then $x^3 = 1$ (always)
- If $x + \frac{1}{x} = \sqrt{3}$, then $x^6 = -1$ or $x^3 + \frac{1}{x^3} = 0$
- If $x + \frac{1}{x} = \sqrt{2}$, then $x^4 = -1$ or $x^2 + \frac{1}{x^2} = 0$
- $a = 500, b = 502, c = 504$ [If a, b, c are in A.P.]
then $a^3 + b^3 + c^3 - 3abc = 9 \times \text{Middle term} \times (\text{Common diff.})^2$
 $= 9 \times 502 \times 2^2$

Linear Equation

- for two linear equation

$$a_1x + b_1y + c_1 = 0 \quad \text{and} \quad a_2x + b_2y + c_2 = 0$$

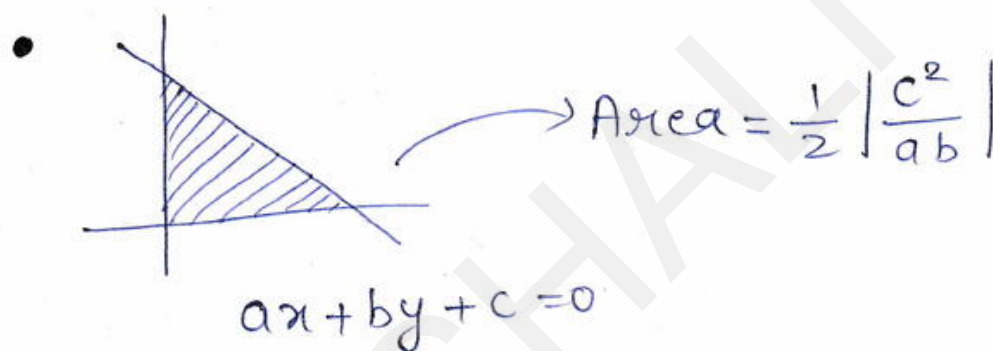
If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \rightarrow$ Only one solution



If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \rightarrow \infty$ solution



If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \rightarrow$ No solution



Quadratic Equation

- $ax^2 + bx + c = 0$ where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a} \quad \left[\text{where } \alpha, \beta \text{ are roots of quadratic equation} \right]$$

- If α, β are roots of quadratic equation

then quadratic equation :- $(x - \alpha)(x - \beta) = 0$

$$= x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

- Nature of roots:-

If $b^2 - 4ac > 0 \rightarrow$ Real and different

If $b^2 - 4ac = 0 \rightarrow$ Real and Equal

If $b^2 - 4ac < 0 \rightarrow$ Imaginary and different

- Maximum and Minimum Value:-

- For $ax^2 + bx + c$

If $a > 0$, minimum = $\frac{4ac - b^2}{4a}$, Maximum = ∞

If $a < 0$, minimum = $-\infty$, Maximum = $\frac{4ac - b^2}{4a}$

- For $ax^2 + \frac{b}{x^2}$

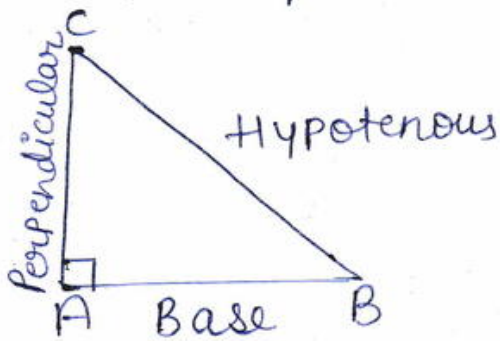
If $a > 0, b > 0$, then minimum = $2\sqrt{ab}$

If $x > 0$, then $x + \frac{1}{x} \geq 2$

If $x < 0$, then $x + \frac{1}{x} \leq -2$

Trigonometry

- $\text{Base}^2 + \text{Perpendicular}^2 = \text{Hypotenous}^2$

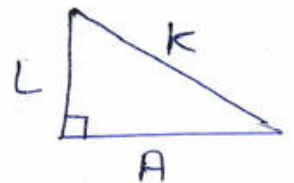


$$(AB)^2 + (AC)^2 = (BC)^2$$

Trigonometry ratio:-

$$\sin \theta = \frac{L}{K}, \cos \theta = \frac{A}{K}, \tan \theta = \frac{L}{A}$$

$$\cot \theta = \frac{A}{L}, \sec \theta = \frac{K}{A}, \csc \theta = \frac{K}{L}$$



$$\sin \theta = \frac{1}{\csc \theta}, \cos \theta = \frac{1}{\sec \theta}, \tan \theta = \frac{1}{\cot \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$\sin \theta \times \csc \theta = 1$
$\cos \theta \times \sec \theta = 1$
$\tan \theta \times \cot \theta = 1$

Complementary Ratio:-

$$\sin (90^\circ - \theta) = \cos \theta, \tan (90^\circ - \theta) = \cot \theta$$

$$\sec (90^\circ - \theta) = \csc \theta, \cos (90^\circ - \theta) = \sin \theta$$

$$\cot (90^\circ - \theta) = \tan \theta, \csc (90^\circ - \theta) = \sec \theta$$

Trigonometry Table:-

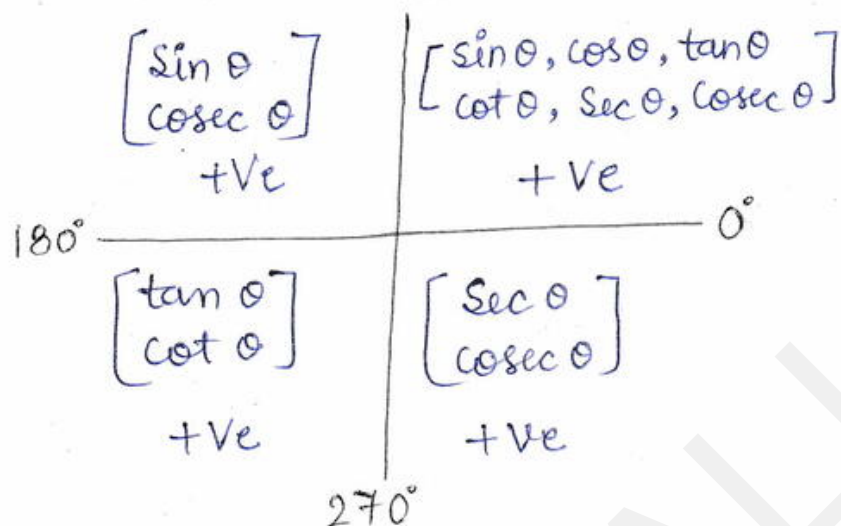
	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cot \theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\operatorname{cosec} \theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Trigonometric Identities:-

- $\sin^2 \theta + \cos^2 \theta = 1$
- $\sec^2 \theta - \tan^2 \theta = 1$
- $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$
- $\sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cos^2 \theta$
- $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$
- $\cot^2 \theta - \cos^2 \theta = \cot^2 \theta \cos^2 \theta$
- If $a \sin \theta + b \cos \theta = c$
 $a \cos \theta - b \sin \theta = d$ then, $a^2 + b^2 = c^2 + d^2$
- If $a \sec \theta + b \tan \theta = c$
 $a \tan \theta + b \sec \theta = d$ then, $a^2 - b^2 = c^2 - d^2$
- If $a \cot \theta + b \operatorname{cosec} \theta = c$
 $a \operatorname{cosec} \theta + b \cot \theta = d$ then $a^2 - b^2 = c^2 - d^2$

- If $\sin \theta + \cos \theta = x$, then $\sin \theta - \cos \theta = \sqrt{2-x^2}$
- If $\sin \theta - \cos \theta = x$, then $\sin \theta + \cos \theta = \sqrt{2-x^2}$
- If $\sec \theta + \tan \theta = x$, then $\sec \theta - \tan \theta = \frac{1}{x}$
- If $\operatorname{cosec} \theta + \cot \theta = x$, then $\operatorname{cosec} \theta - \cot \theta = \frac{1}{x}$

90° [Quadrant Theory]



- $\sin(-\theta) = -\sin \theta$, $\tan(-\theta) = -\tan \theta$
 $\cos(-\theta) = \cos \theta$, $\cot(-\theta) = -\cot \theta$
 $\sec(-\theta) = \sec \theta$
- $\sin(90^\circ - \theta) = \cos \theta$, $\cos(90^\circ - \theta) = \sin \theta$
 $\tan(90^\circ - \theta) = \cot \theta$, $\cot(90^\circ - \theta) = \tan \theta$
 $\sec(90^\circ - \theta) = \operatorname{cosec} \theta$, $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$
- $\sin(90^\circ + \theta) = \cos \theta$, $\cos(90^\circ + \theta) = -\sin \theta$
 $\tan(90^\circ + \theta) = -\cot \theta$, $\cot(90^\circ + \theta) = -\tan \theta$
 $\sec(90^\circ + \theta) = -\operatorname{cosec} \theta$, $\operatorname{cosec}(90^\circ + \theta) = \sec \theta$

- $\sin(180^\circ - \theta) = \sin \theta$, $\cos(180^\circ - \theta) = -\cos \theta$
 $\cot(180^\circ - \theta) = -\cot \theta$, $\tan(180^\circ - \theta) = -\tan \theta$
 $\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec} \theta$, $\sec(180^\circ - \theta) = -\sec \theta$
- $\sin(180^\circ + \theta) = -\sin \theta$, $\cos(180^\circ + \theta) = -\cos \theta$
 $\tan(180^\circ + \theta) = \tan \theta$, $\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec} \theta$
 $\sec(180^\circ + \theta) = -\sec \theta$, $\cot(180^\circ + \theta) = \cot \theta$
- $\sin(270^\circ - \theta) = -\cos \theta$, $\cos(270^\circ - \theta) = -\sin \theta$
 $\tan(270^\circ - \theta) = \cot \theta$, $\operatorname{cosec}(270^\circ - \theta) = -\sec \theta$
 $\sec(270^\circ - \theta) = -\operatorname{cosec} \theta$, $\cot(270^\circ - \theta) = \tan \theta$
- $\sin(270^\circ + \theta) = -\cos \theta$, $\cos(270^\circ + \theta) = \sin \theta$
 $\tan(270^\circ + \theta) = -\cot \theta$, $\operatorname{cosec}(270^\circ + \theta) = -\sec \theta$
 $\sec(270^\circ + \theta) = \operatorname{cosec} \theta$, $\cot(270^\circ + \theta) = -\tan \theta$
- If $A + B = 90^\circ$, then
 - ★ $\sin^2 A + \sin^2 B = 1$
 - ★ $\cos^2 A + \cos^2 B = 1$
 - ★ $\sin A \cdot \sec B = 1$
 - ★ $\cos A \cdot \operatorname{cosec} B = 1$
 - ★ $\tan A \cdot \tan B = 1$
 - ★ $\cot A \cdot \cot B = 1$
 - ★ $\sin A = \cos B$
 - ★ $\tan A = \cot B$
 - ★ $\operatorname{cosec} A = \sec B$

- If $\cos^2 \theta + \cos^4 \theta = 1$, then $\tan^2 \theta + \tan^4 \theta = 1$
- If $\sin^2 \theta + \sin^4 \theta = 1$, then $\cot^2 \theta + \cot^4 \theta = 1$
- If $\cot^4 \theta - \cot^2 \theta = 1$, then $\sec^4 \theta - \sec^2 \theta = 1$
- If $\sin \theta + \sin^2 \theta = 1$, then $\cos^2 \theta + \cos^4 \theta = 1$

formula:-

- $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- $\sin(A-B) = \sin A \cos B - \cos A \sin B$
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\cos(A-B) = \cos A \cos B + \sin A \sin B$
- $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
- $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$
 $= \frac{1 - \tan^2 A}{1 + \tan^2 A}$
- $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$
- $\sin 3A = 3 \sin A - 4 \sin^3 A$
- $\cos 3A = 4 \cos^3 A - 3 \cos A$

- $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$
- $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$
- $\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$
- $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$
- $\cos C - \cos D = 2 \sin \frac{C+D}{2} \sin \frac{D-C}{2}$
- $\sin \theta \sin (60^\circ - \theta) \sin (60^\circ + \theta) = \frac{\sin 3\theta}{4}$
- $\cos \theta \cos (60^\circ - \theta) \cos (60^\circ + \theta) = \frac{\cos 3\theta}{4}$
- $\tan \theta \tan (60^\circ - \theta) \tan (60^\circ + \theta) = \tan 3\theta$
- $\sin \theta \sin 2\theta \sin 4\theta = \frac{1}{4} \sin 3\theta$
- $\cos \theta \cos 2\theta \cos 4\theta = \frac{1}{4} \cos 3\theta$
- $\tan \theta \tan 2\theta \tan 4\theta = \tan 3\theta$

Degree & Radian

$$\pi \text{ Radian} = 180^\circ$$

$$1 \text{ Radian} = \frac{180^\circ}{\pi} = 57^\circ 16' 22''$$

$$1^\circ = \frac{\pi}{180} \text{ Radian} = \left(\frac{11}{630}\right)$$

Maxima & Minima

$$\sin \theta, \cos \theta = [-1, 1], \quad \sin^2 \theta, \cos^2 \theta = [0, 1]$$

$$\tan \theta, \cot \theta = (-\infty, \infty), \quad \tan^2 \theta, \cot^2 \theta = (0, \infty)$$

$$\sec \theta, \operatorname{cosec} \theta = (-\infty, \infty) \text{ [but not between } -1 \text{ and } 1]$$

$$\sec^2 \theta, \operatorname{cosec}^2 \theta = [1, \infty]$$

Type-1:- $a \sin \theta + b \cos \theta$

$$\max = \sqrt{a^2 + b^2}, \quad \min = -\sqrt{a^2 + b^2}$$

Type-2:- $a \cos^2 \theta + b \sin^2 \theta$

$$\max = \max \text{ in } a \text{ and } b$$

$$\min = \min \text{ in } a \text{ and } b$$

Type-3:- $\sin^4 \theta + \cos^4 \theta$ OR $\sin^6 \theta + \cos^6 \theta$

$$\max = 1, \quad \min = \frac{1}{2}$$

$$\boxed{AM \geq GM}$$

- $x^2 + \frac{1}{x^2} \geq 2$
- $\tan^2 \theta + \cot^2 \theta \geq 2$
- $\sin^2 \theta + \operatorname{cosec}^2 \theta \geq 2$
- $\cos^2 \theta + \sec^2 \theta \geq 2$

Type-4:- $a \tan^2 \theta + b \cot^2 \theta$

$$\text{Min} = 2\sqrt{ab}, \quad \text{Max} = \infty$$

Type-5:- $a^2 \sec^2 \theta + b^2 \operatorname{cosec}^2 \theta$

$$\text{min} = (a+b)^2, \quad \text{max} = \infty$$

Important formula:-

- $\tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta}$
- $\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$
- $\operatorname{cosec}^2 \theta + \sec^2 \theta = \operatorname{cosec}^2 \theta \times \sec^2 \theta$
- $(\sin \theta + \cos \theta)^2 = 1 + \sin 2\theta$
- $(\sin \theta - \cos \theta)^2 = 1 - \sin 2\theta$
- If $A+B = 45^\circ$ or 225° , then
$$(1 + \tan A)(1 + \tan B) = 2$$
$$(\cot A - 1)(\cot B - 1) = 2$$
$$(1 - \cot A)(1 - \cot B) = 2$$
- $\tan (A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$
- If $A+B+C = 90^\circ$
then, $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$
OR $\cot A + \cot B + \cot C = \cot A \cot B \cot C$

- If $A + B + C = 180^\circ$

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C \cdot \cot A = 1$$

- $\left[\frac{1 - \tan \theta}{1 - \cot \theta} \right]^2 = \tan^2 \theta$

- $\sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \frac{1 - \sin \theta}{\cos \theta} = \sec \theta - \tan \theta$

- $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = \frac{1 + \cos \theta}{\sin \theta} = \operatorname{cosec} \theta + \cot \theta$

- $\sqrt{\frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}} = \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

- $\sqrt{\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta}} = \sqrt{(\sec \theta + \tan \theta)^2} = \sec \theta + \tan \theta$

- $\sqrt{\frac{\cot \theta + \cos \theta}{\cot \theta - \cos \theta}} = \sec \theta + \tan \theta$

- $\sin 1^\circ \cdot \sin 2^\circ \cdot \sin 3^\circ \dots \sin 180^\circ = 0$

- $\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \dots \cos 90^\circ = 0$

- $\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 89^\circ = 1$

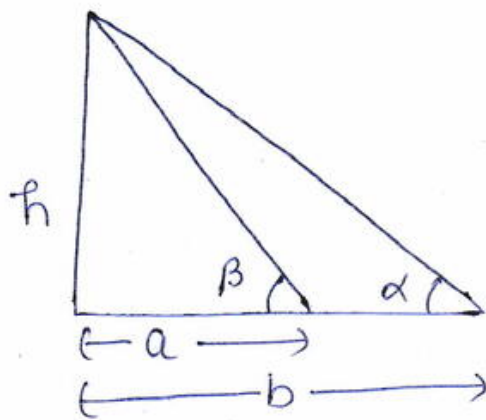
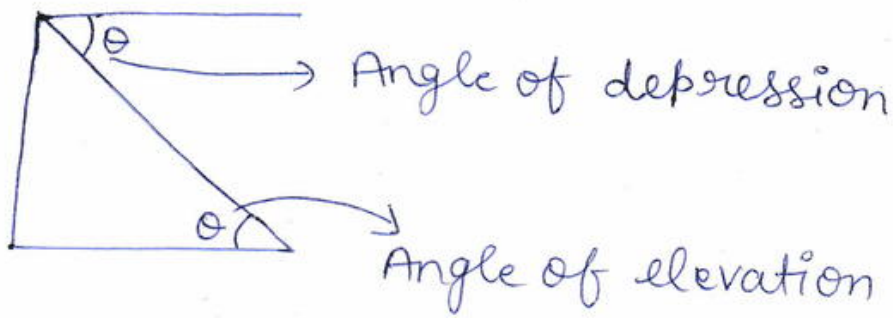
- $\sin 75^\circ = \cos 15^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$

- $\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$

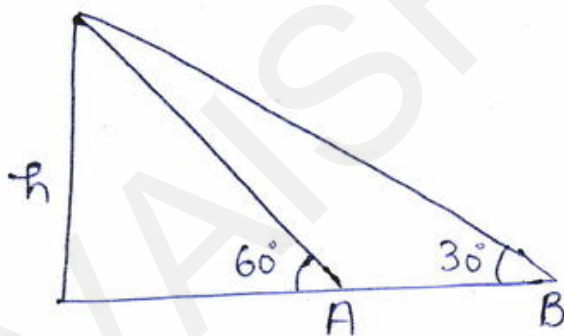
- $\tan 75^\circ = \cot 15^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3}$

- $\tan 15^\circ = \cot 75^\circ = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$

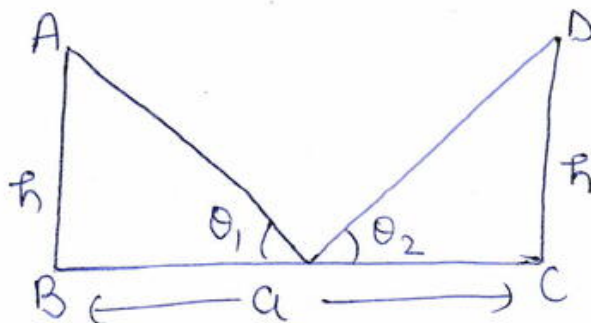
Height and Distance



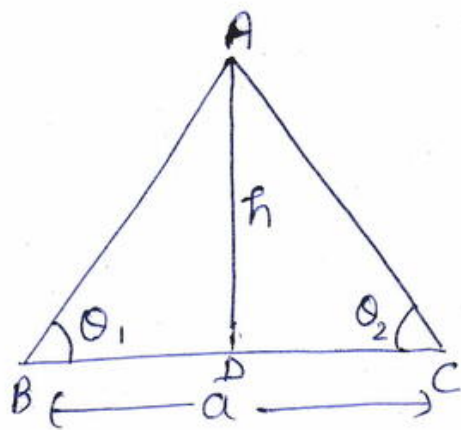
If $\alpha + \beta = 90^\circ$
then Height $= \sqrt{ab}$



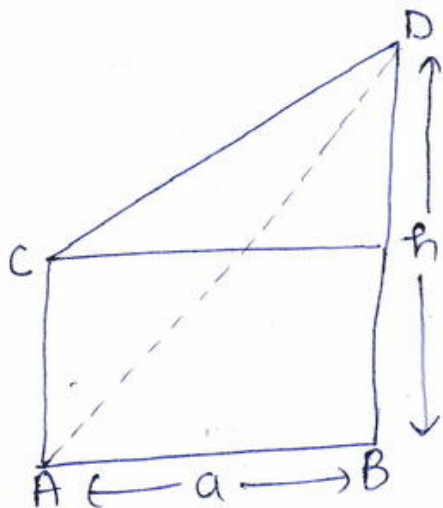
If $\alpha = 30^\circ$, $\beta = 60^\circ$
then $AB = \frac{2h}{\sqrt{3}}$



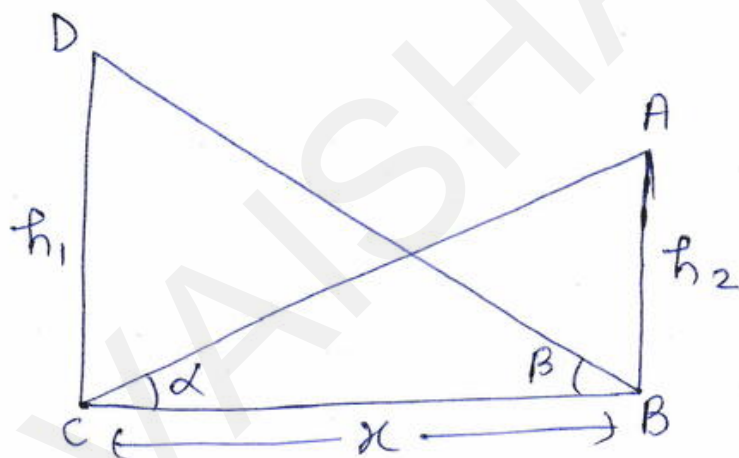
$$a = h (\cot \theta_1 + \cot \theta_2)$$



$$a = h (\cot \theta_1 + \cot \theta_2)$$



$$h = \frac{a \cot \theta_1}{\cot \theta_1 - \cot \theta_2}$$



$$\text{If } \alpha + \beta = 90^\circ$$

$$x = \sqrt{h_1 h_2}$$

Geometry

• Line:- 

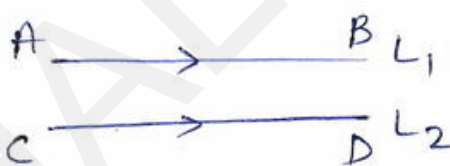
• Line Segment:- 

• Ray:- 

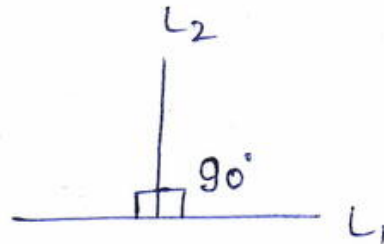
• Collinear Points:- If 3 or more than 3 points are on the line, they are collinear.



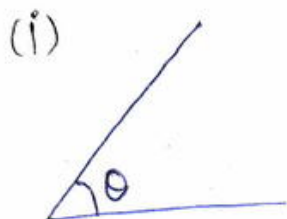
A, B and C are collinear.

• Parallel lines:-  $AB \parallel CD$

• Perpendicular lines:-

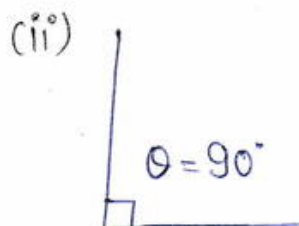


Types of Angles:-



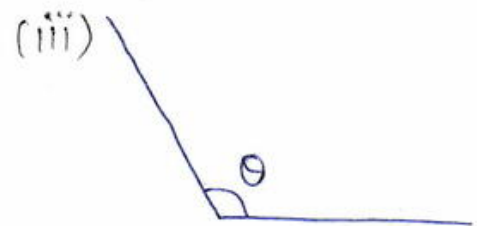
Acute Angle

$$0^\circ < \theta < 90^\circ$$



Right Angle

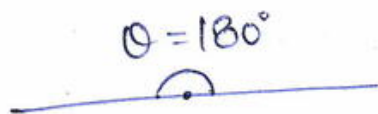
$$\theta = 90^\circ$$



Obtuse Angle

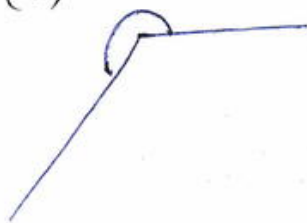
$$180^\circ > \theta > 90^\circ$$

(iv)



Straight Angle
 $\theta = 180^\circ$

(v)



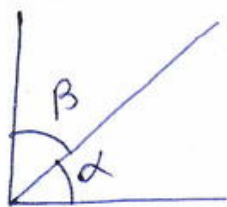
Reflex Angle
 $180^\circ < \theta < 360^\circ$

(vi)



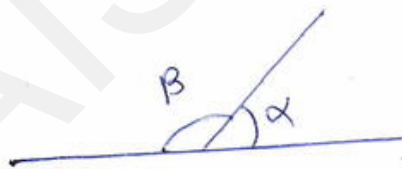
Complete Angle
 $\theta = 360^\circ$

(vii) Complementary Angle:- If sum of two angles is 90° then they are complementary to each other.



$$\alpha + \beta = 90^\circ$$

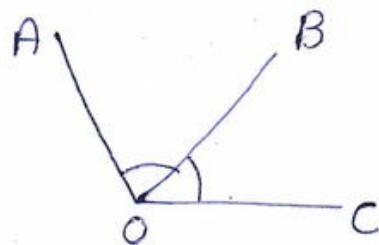
(viii) Supplementary Angles:- If sum of two angles is 180° then they are supplementary to each other.



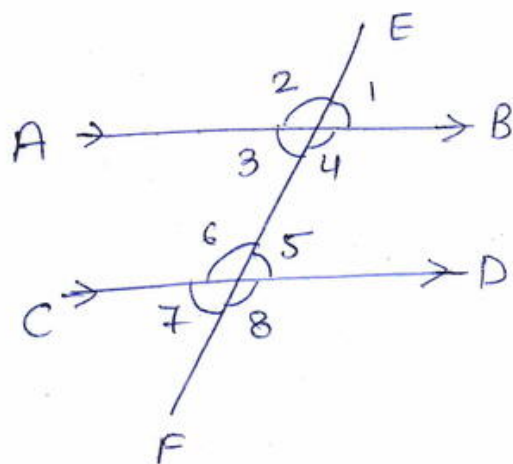
$$\alpha + \beta = 180^\circ$$

(ix) Adjacent Angle:-

$\angle AOB$ and $\angle BOC$ are adjacent angles.



- $AB \parallel CD$ and EF is transversal line



(x) Corresponding Angles:- $\angle 1 = \angle 5$, $\angle 4 = \angle 8$
 $\angle 3 = \angle 7$, $\angle 2 = \angle 6$

(xi) Alternate Interior Angles:- $\angle 3 = \angle 5$, $\angle 4 = \angle 6$

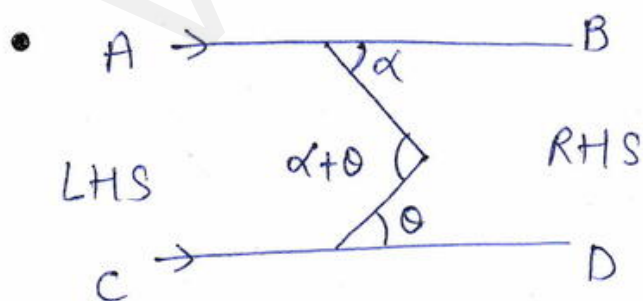
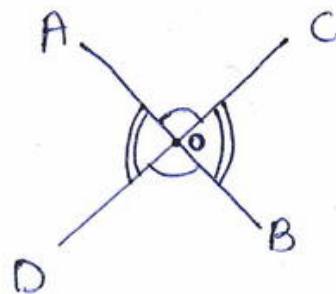
$$\star \angle 4 + \angle 5 = 180^\circ$$

$$\angle 3 + \angle 6 = 180^\circ$$

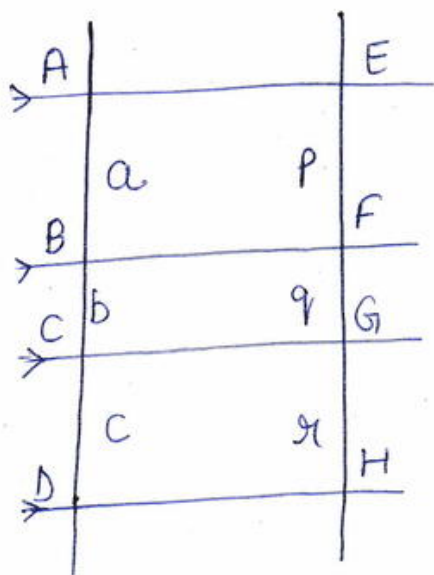
(xii) Vertically opposite Angle:-

$$\angle AOC = \angle DOB$$

$$\angle AOD = \angle COB$$

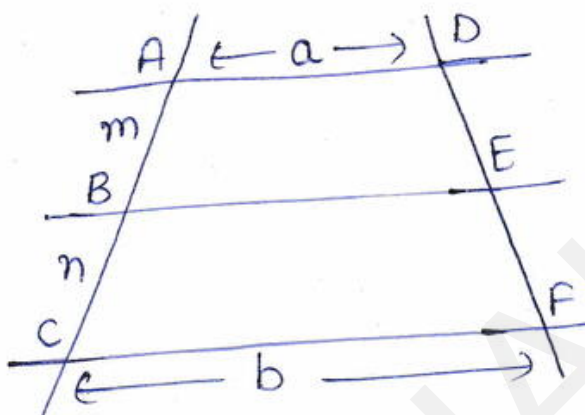


Sum of angle on RHS = LHS



$$a:b:c = p:q:r$$

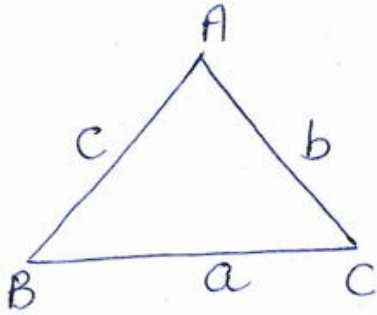
$$\frac{a}{a+b+c} = \frac{p}{p+q+r}$$



$$\frac{AB}{BC} = \frac{DE}{EF} = \frac{m}{n}$$

$$BE = \frac{an+bm}{m+n}$$

Triangle



$$\angle A + \angle B + \angle C = 180^\circ$$

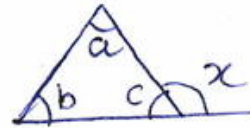
[3 sides, 3 vertices, 3 angles,
3 altitudes]

$$\begin{array}{l|l} a+b > c & a-b < c \\ b+c > a & b-c < a \\ c+a > b & c-a < b \end{array}$$

• Exterior Angle:-

Exterior angle = sum of interior opposite angle

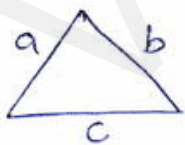
$$\angle x = \angle a + \angle b$$



Types of Triangle:-

Based on angles:-

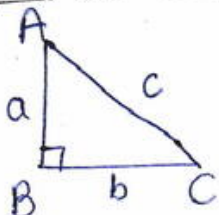
• Acute angle Triangle:-



All three angles $< 90^\circ$

$$\text{Largest side} \Rightarrow a^2 + b^2 > c^2$$

• Right angle Triangle:-



One angle is 90°

$$\angle B = 90^\circ$$

$$\angle A + \angle C = 90^\circ$$

$$a^2 + b^2 = c^2$$

• Obtuse angle Triangle:-



One angle $> 90^\circ$
 $a^2 + b^2 < c^2$

Based on sides:-

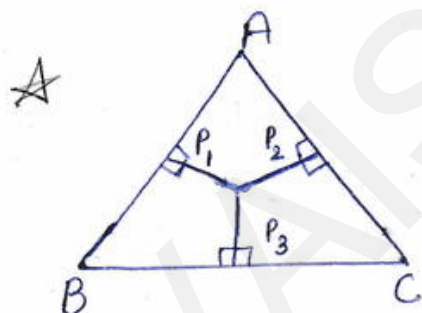
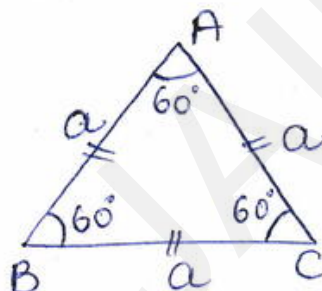
- Equilateral Triangle:- All 3 sides are equal, each angle $= 60^\circ$

$$\text{Area} = \frac{\sqrt{3}}{4} a^2$$

$$\text{Height} = \frac{\sqrt{3}}{2} a$$

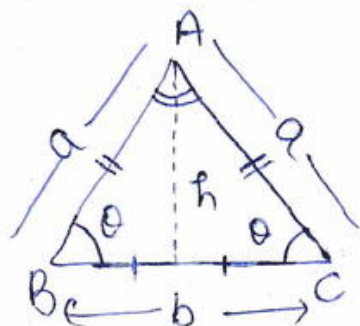
$$\text{Circum Radius} = \frac{a}{\sqrt{3}}$$

$$\text{Inradius} = \frac{a}{2\sqrt{3}}$$



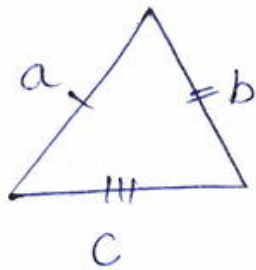
$$h = p_1 + p_2 + p_3$$

- Isosceles Triangle:- two sides are equal, two angles are equal



$$h = \sqrt{b^2 - \frac{a^2}{4}}$$

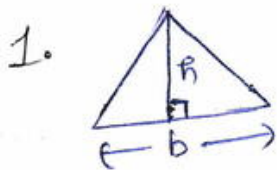
- scalene Triangle :- All sides are different



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

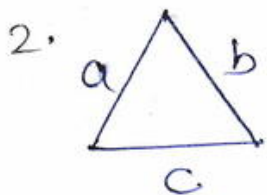
$$s = \frac{a+b+c}{2}$$

Area of Triangle :-



$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{height}$$

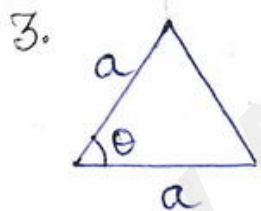
$$\Rightarrow \frac{1}{2} \times b \times h$$



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

(Heron's formula)



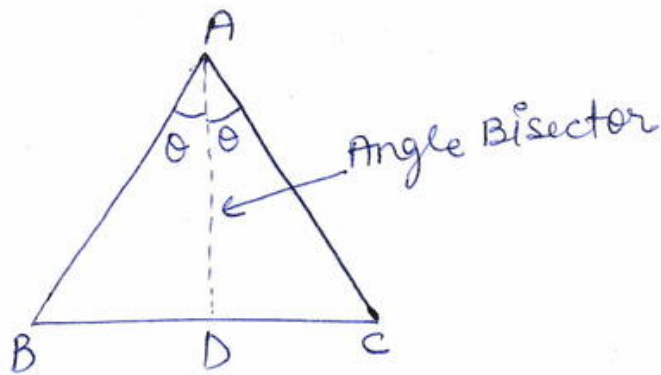
$$\text{Area} = \frac{1}{2} ab \sin \theta$$

4. If the medians m_1, m_2, m_3 is given,

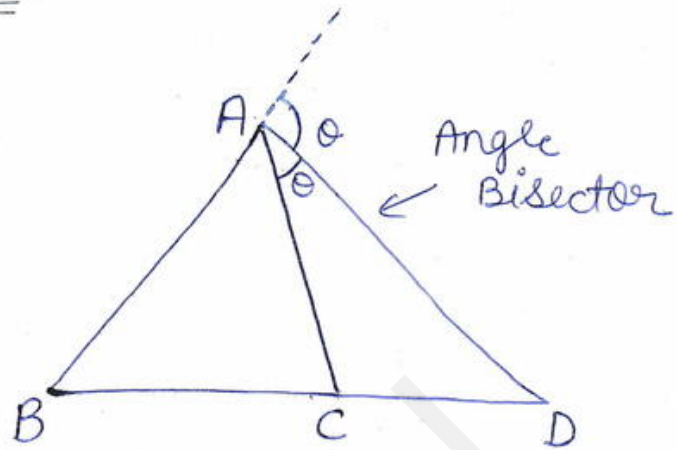
$$\text{Area of Triangle} = \frac{4}{3} \sqrt{s(s-m_1)(s-m_2)(s-m_3)}$$

$$s = \frac{m_1+m_2+m_3}{2}$$

• Angle Bisector Theorem:-



$$\frac{BD}{DC} = \frac{AB}{AC}$$

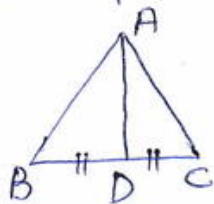


$$\frac{BD}{DC} = \frac{AB}{AC}$$

- Angle Bisector:- A line that splits an angle into two equal angles.

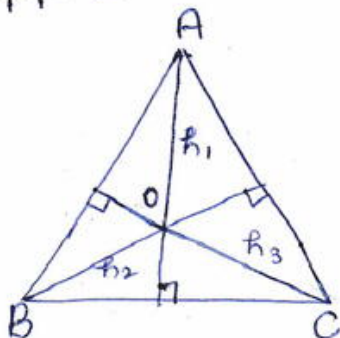


- Median:- Line drawn from a vertex to opposite side which divides the opposite side into equal parts.



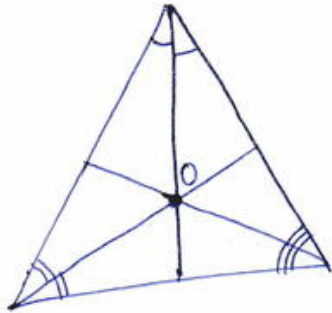
AD is the median of side BC.

- Perpendicular bisector:- The line segment that is drawn from a vertex to the opposite side bisecting it at right angle.



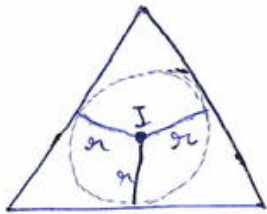
O = Circumcentre

- Incentre :- Incentre is the intersection point of all three internal angle bisectors of $\triangle ABC$.



O is the incentre.

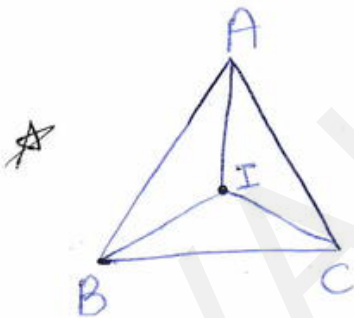
- * Incentre is equidistant from all three sides of triangle



$$\text{Radius } (r) = \frac{\Delta}{S}$$

Δ = Area of Triangle

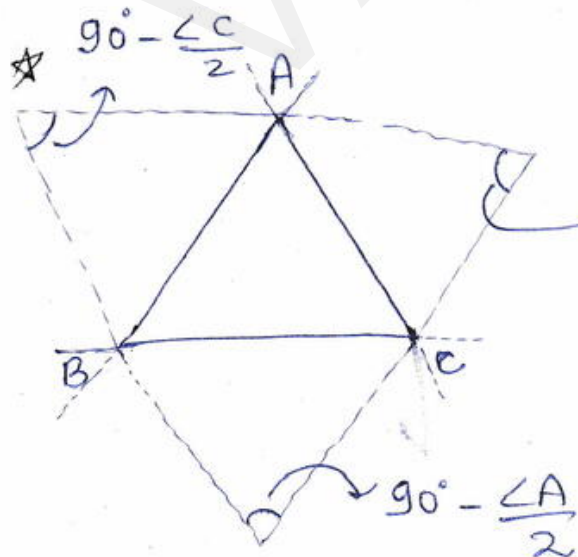
$$S = \frac{a+b+c}{2}$$



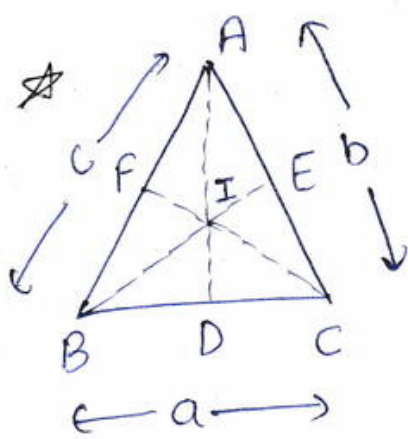
$$\angle BIC = 90^\circ + \frac{\angle A}{2}$$

$$\angle AIC = 90^\circ + \frac{\angle B}{2}$$

$$\angle AIB = 90^\circ + \frac{\angle C}{2}$$



Angles on excentre

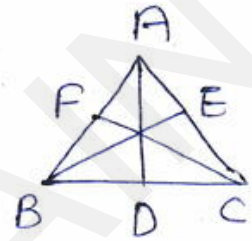


$$\frac{AI}{ID} = \frac{b+c}{a}$$

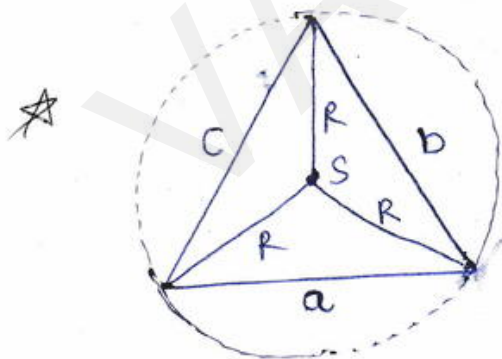
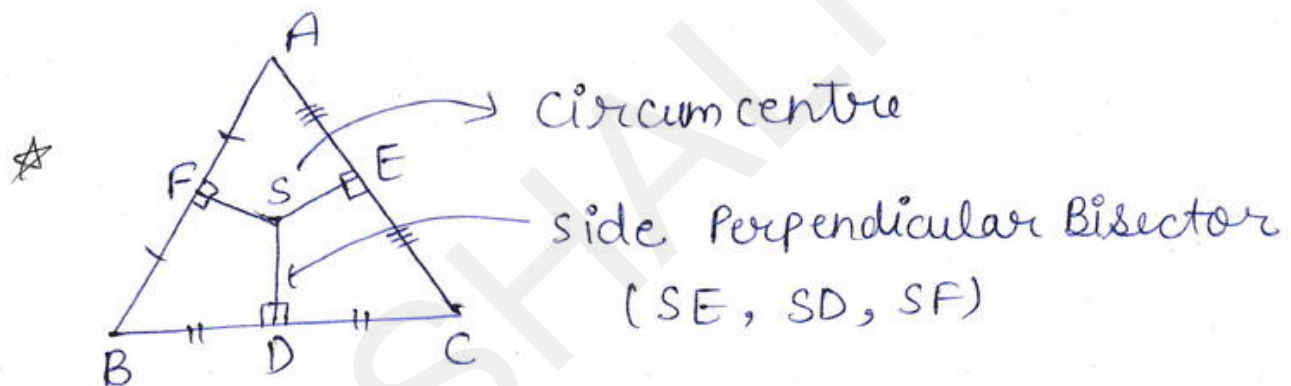
$$\frac{CI}{IF} = \frac{b+a}{c}$$

$$\frac{BI}{IE} = \frac{a+c}{b}$$

* length of angle bisector
 $AD^2 = AB \times AC - BD \times CD$



• Circumcentre / Circumcircle:-

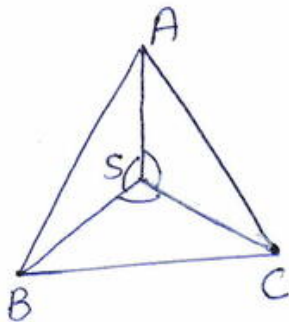


S is centre of circumcircle

$$\text{Circum Radius } (R) = \frac{abc}{4\Delta}$$

$\Delta \rightarrow$ Area of Triangle

★ Angle on circumcentre :-



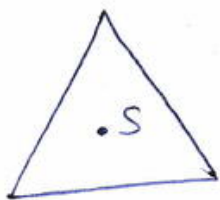
$$\angle BSC = 2\angle A$$

$$\angle ASC = 2\angle B$$

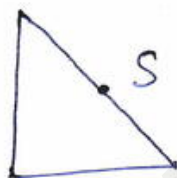
$$\angle ASB = 2\angle C$$

★ S (Circumcentre) is equal distance from vertices.

★ Points of circumcentre :-



Acute Δ

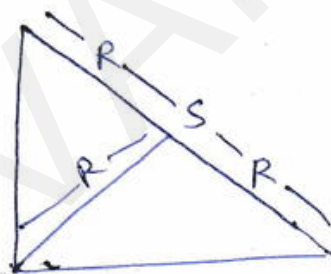


In Right angle Δ



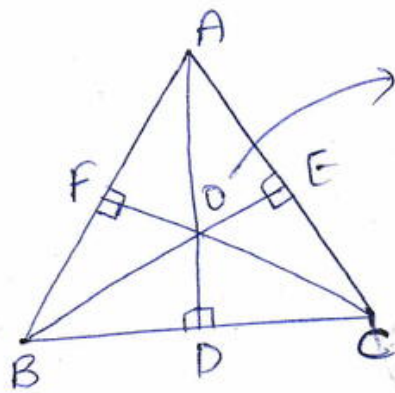
In obtuse Angle Δ

★ In right angle Δ :-



$$\text{Circum Radius (R)} = \frac{\text{Hypotenous}}{2}$$

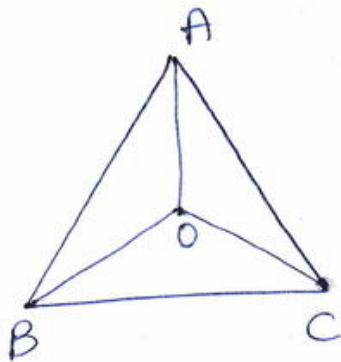
• Orthocentre :-



Orthocentre

AD, BE & CF are altitude.

★ Angles on Orthocentre :-

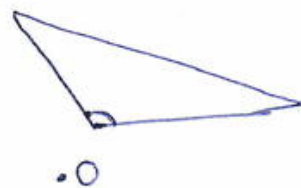
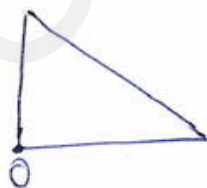
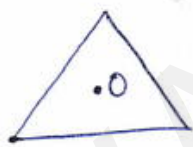


$$\angle BOC = 180^\circ - \angle A$$

$$\angle AOC = 180^\circ - \angle B$$

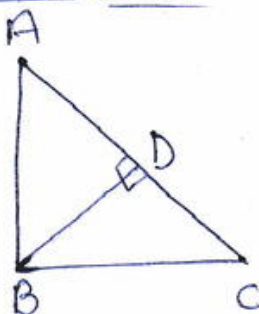
$$\angle AOB = 180^\circ - \angle C$$

★ Position of Orthocentre :-



Acute angle Δ Right angle Δ Obtuse angle Δ

★ Right angle Δ :-

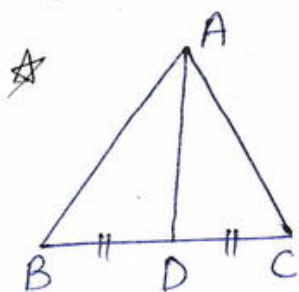


$$BD = \frac{AB \times BC}{AC}$$

$$BD^2 = AD \times CD$$

$$AD = \frac{AB^2}{AC}, \quad CD = \frac{BC^2}{AC}$$

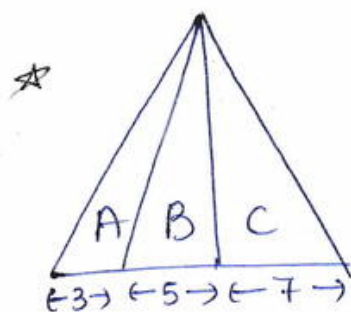
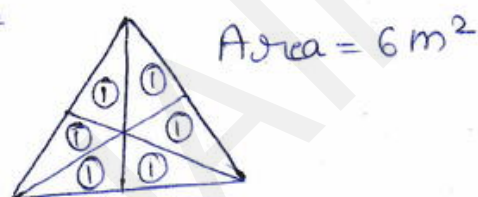
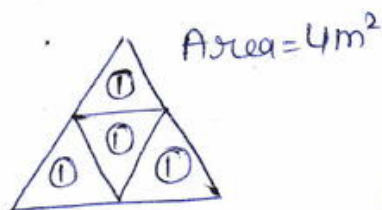
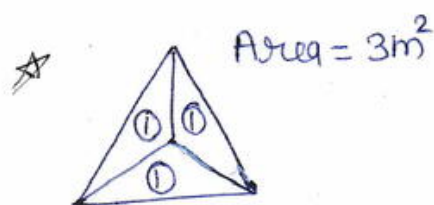
• Median:-



AD is the median of $\triangle ABC$

$$BD = DC$$

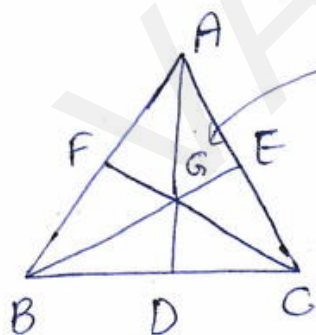
★ Median theorem:- $AB^2 + AC^2 = 2(AD^2 + DC^2)$



$A : B : C$
Area Ratio = $3 : 5 : 7$

Ratio of area is equal to the ratio of base sides

• Centroid:-



centroid

AD, BE and CF are medians

$$\star \frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$$

$$\star [3(AB^2 + BC^2 + CA^2)] = [4(AD^2 + BE^2 + CF^2)]$$

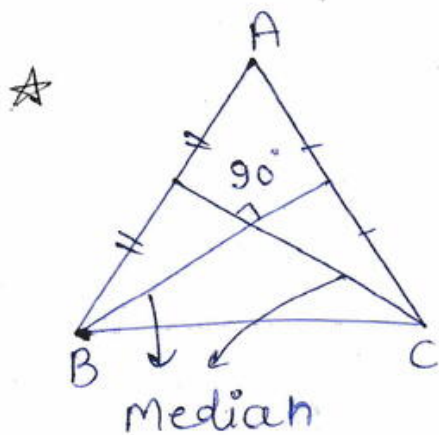
$$\star 4(m_1 + m_2 + m_3) > 3(AB + BC + CA)$$

m_1, m_2 & m_3 are medians

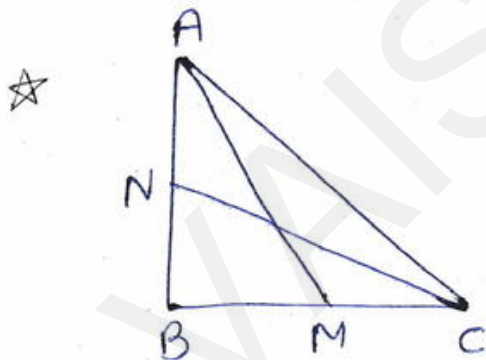
$$\text{Area of triangle} = \frac{4}{3} (\sqrt{S(S-m_1)(S-m_2)(S-m_3)})$$

$$\text{where, } S = \frac{m_1 + m_2 + m_3}{2}$$

$$\text{If } m_1 \triangle m_3, \text{ then area} = \frac{4}{3} \left[\frac{1}{2} \times m_1 \times m_2 \right]$$



$$AB^2 + AC^2 = 5BC^2$$



$$AM^2 + CN^2 = \frac{5}{4} AC^2$$

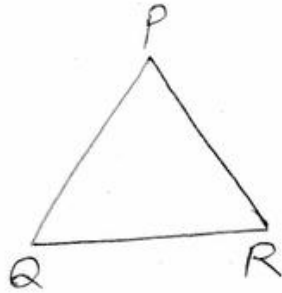
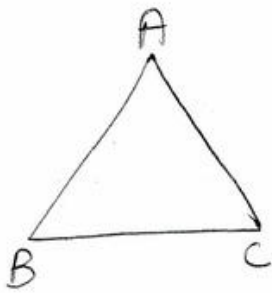
Where AM & CN are median

$\star$ Distance between incentre & circum centre

$$= \sqrt{R^2 - 2Rr}$$

Where, R = Circum Centre
 r = Inradius

Congruence



$$\triangle ABC \cong \triangle PQR$$

$$\text{If } AB = PQ, BC = QR,$$

$$CA = RP$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

$\cong \rightarrow$ Congruence

Side - Angle - Side Rule :- (SAS)

$$\text{If } AB = PQ \text{ (Side)}$$

$$\angle B = \angle Q \text{ (Angle)}$$

$$BC = QR \text{ (Side)}$$

$$\text{then } \triangle PQR \cong \triangle ABC$$

Angle - Angle - Side Rule :- (AAS or ASA)

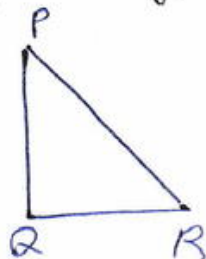
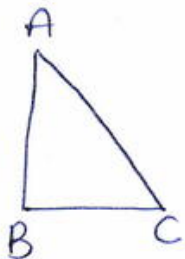
$$\text{If } \angle A = \angle P$$

$$\angle B = \angle Q$$

$$AB = DE$$

$$\text{then } \triangle ABC \cong \triangle PQR$$

Right Angle - Hypotenuse - Side (RHS)



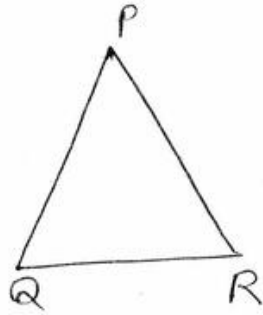
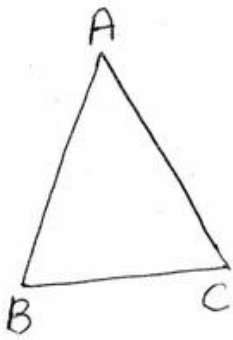
$$\angle B = \angle Q \text{ (Right angle)}$$

$$AC = PQ \text{ (Hypotenuse)}$$

$$AB = PR \text{ (Side)}$$

$$\text{then } \triangle ABC \cong \triangle PQR$$

Similarity



$$\triangle ABC \sim \triangle PQR$$

$$(i) \text{ If } \begin{cases} \angle A = \angle P \\ \angle B = \angle Q \\ \angle C = \angle R \end{cases} \left\{ \begin{array}{l} \text{Corresponding} \\ \text{Angles are} \\ \text{equal} \end{array} \right.$$

$$(ii) \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

Angle - Angle - Angle :- (AAA)

$$\text{If } \begin{aligned} \angle A &= \angle P \\ \angle B &= \angle Q \\ \angle C &= \angle R \end{aligned}$$

then, $\triangle ABC \sim \triangle PQR$

Side - Side - Side :- (SSS)

$$\text{If } \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

then, $\triangle ABC \sim \triangle PQR$

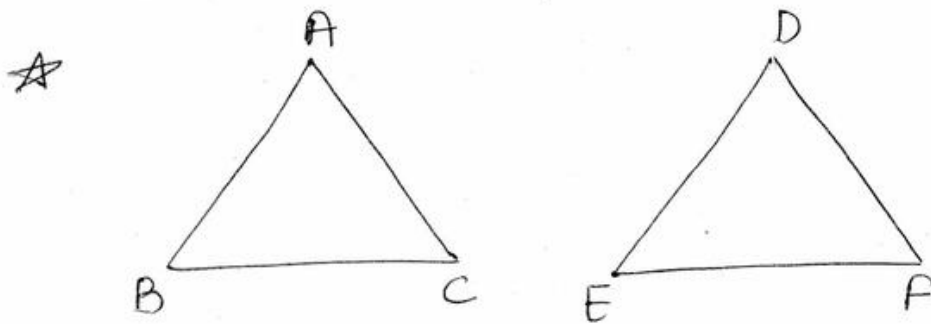
Side - Angle - Side :- (SAS)

$$\angle A = \angle P \text{ (Angle)}$$

$$\frac{AB}{PQ} = \frac{AC}{PR}$$

then $\triangle ABC \sim \triangle PQR$

• In similar Triangles:-



$$\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{A \cdot B_1}{A \cdot B_2}$$

$$= \frac{A \cdot L_1}{A \cdot L_2} = \frac{m_1}{m_2} = \frac{r_1}{r_2} = \frac{R_1}{R_2}$$

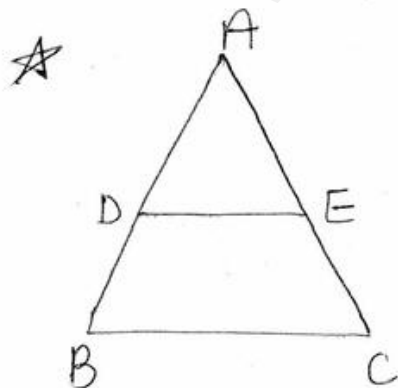
$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = \frac{A \cdot B_1^2}{A \cdot B_2^2}$$

$$= \frac{A \cdot L_1^2}{A \cdot L_2^2} = \frac{m_1^2}{m_2^2} = \frac{r_1^2}{r_2^2} = \frac{R_1^2}{R_2^2}$$

Here, $A \cdot B \rightarrow$ Angle Bisector, $A \cdot L \rightarrow$ Altitude

$m \rightarrow$ median, $r \rightarrow$ Inradius

$R \rightarrow$ Circumradius

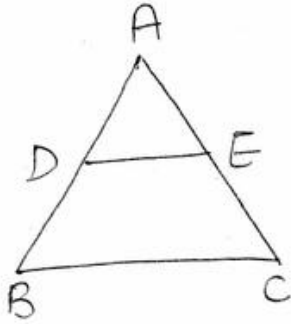


$$\triangle ADE \sim \triangle ABC$$

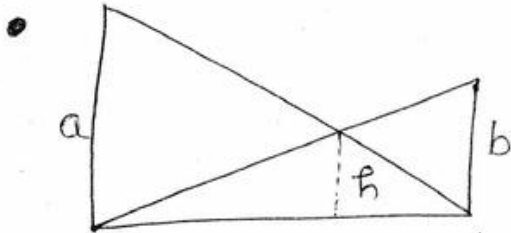
$$\text{If } DE \parallel BC$$

$$\text{then, } \frac{AD}{DB} = \frac{AE}{EC}, \frac{AD}{AB} = \frac{AE}{AC}, \frac{BD}{AB} = \frac{EC}{AC}$$

• Mid-Point Theorem:-

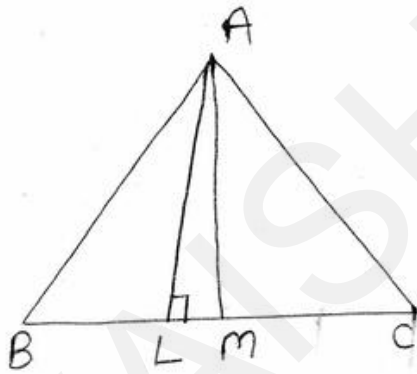


If in $\triangle ABC$ D and E are mid point of AB and AC and $DE \parallel BC$ then, $DE = \frac{BC}{2}$

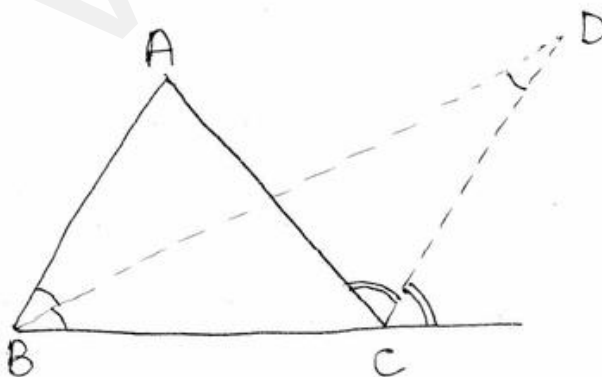


$$h = \frac{ab}{a+b}$$

• Angle between altitude and angle bisector:-



$$\angle MAL = \frac{1}{2} |\angle B - \angle C|$$



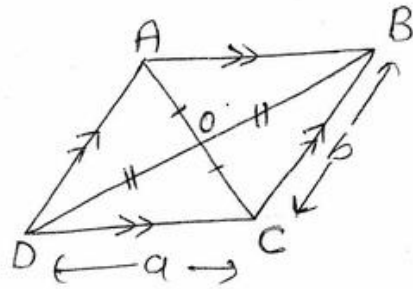
BD and CD are angle bisectors

then $\angle D = \frac{\angle A}{2}$

Quadrilaterals

1. Parallelogram:-

$$\begin{array}{l|l} \star AB \parallel CD & AB = CD \\ AD \parallel BC & AD = BC \end{array}$$



★ Opposite angles are equal

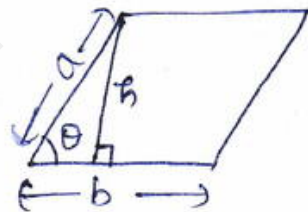
$$\angle B = \angle D, \angle A = \angle C$$

★ Diagonals bisect equally

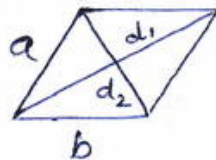
$$OA = OC, OD = OB$$

★ Area = $\frac{1}{2} \times \text{Base} \times \text{height}$

$$= ab \sin \theta$$



$$\star 2(a^2 + b^2) = (d_1^2 + d_2^2)$$



★ Area of $\triangle AOB = \text{Area of } \triangle BOC = \text{Area of } \triangle AOD$
 $= \frac{1}{4} \text{ area of } \square ABCD$

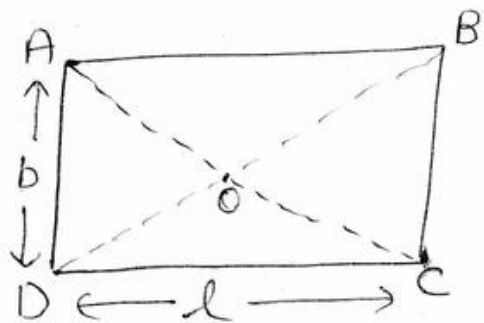
2. Rectangle:-

★ All Angles = 90°

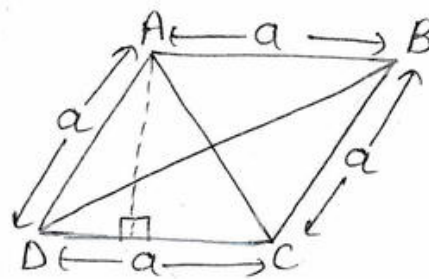
★ $AB = CD, AD = BC$

★ $AO = OC, BO = OD$ (Diagonals bisect equally)

★ Area = $l \times b$



3. Rhombus:-



★ All sides are equal

★ $AB \parallel CD$ and $AD \parallel BC$

★ Opposite angles are equal

$$\angle A = \angle C, \angle B = \angle D$$

★ BD and AC are angle bisector

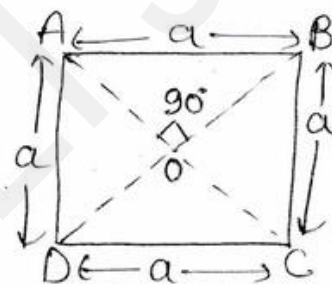
★ Area = base $\times$ height

$$\text{Area} = \frac{1}{2} \times d_1 \times d_2$$

[d_1 and d_2 are diagonals]

4. Square:-

All sides are equal
every angle = 90°



$$\text{Area} = (\text{Side})^2 = a^2$$

$$\text{Diagonals} = \sqrt{2} a$$

$$\text{In radius of square} = \frac{a}{2}$$



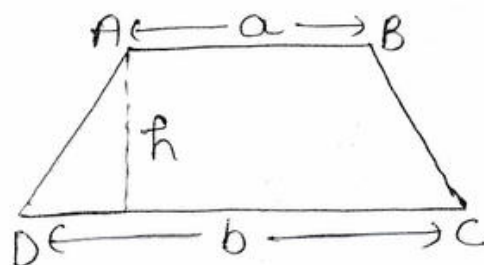
$$\text{Circum Radius of square} = \frac{a}{\sqrt{2}}$$



5. Trapezium:-

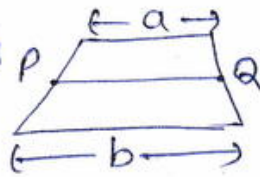
If $AB \parallel CD$

and $AD \nparallel BC$



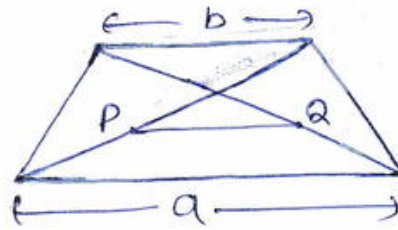
$$\text{★ Area} = \frac{1}{2} \times (a + b) \times h$$

★ P & Q are mid-points $PQ = \frac{1}{2}(a+b)$



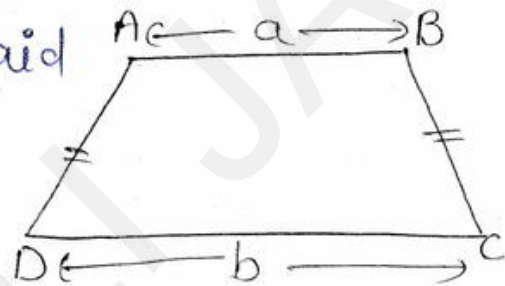
★ $PQ = \frac{a-b}{2}$

[Here, P and Q are mid points]



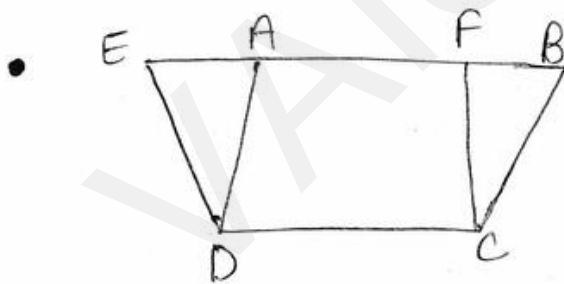
6. Isosceles Trapezium:-

If $AD = BC$, then it is said to be Isosceles Trapezium

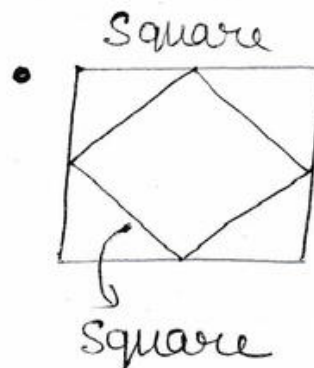
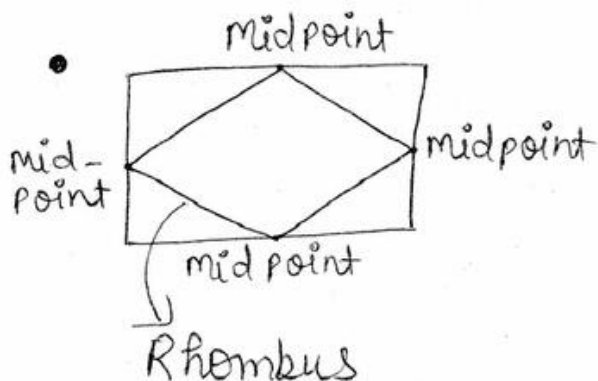


then,
 $\angle A = \angle B$
 $\angle C = \angle D$

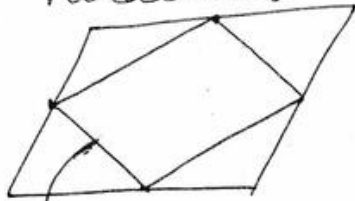
Important points of Quadrilateral:-



Area of ABCD = Area of CDEF
 $EB \parallel CD$

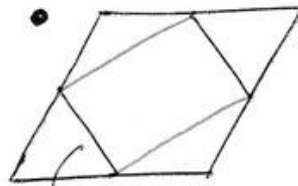


• Parallelogram



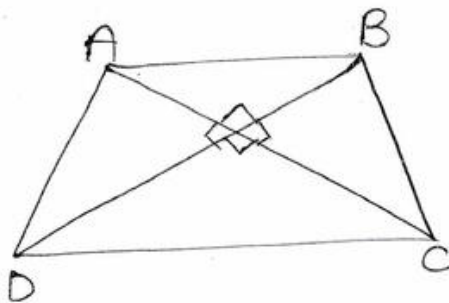
Parallelogram

• Rhombus

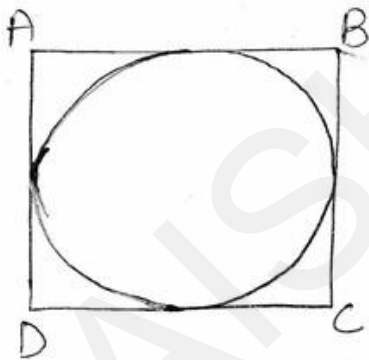


Rectangle

- If diagonal intersect on 90°

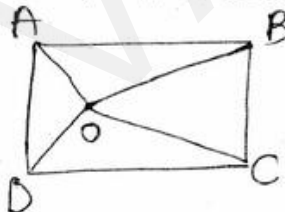


$$AB^2 + CD^2 = BC^2 + AD^2$$



$$AB + CD = BC + AD$$

- In Rectangle



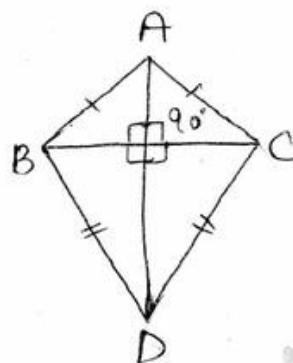
$$OA^2 + OC^2 = OB^2 + OD^2$$

- Kite :-

$$AB = AC$$

$$BD = DC$$

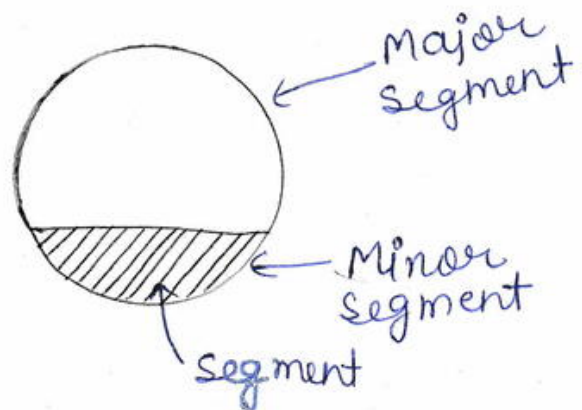
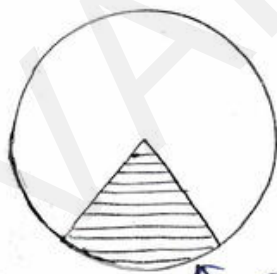
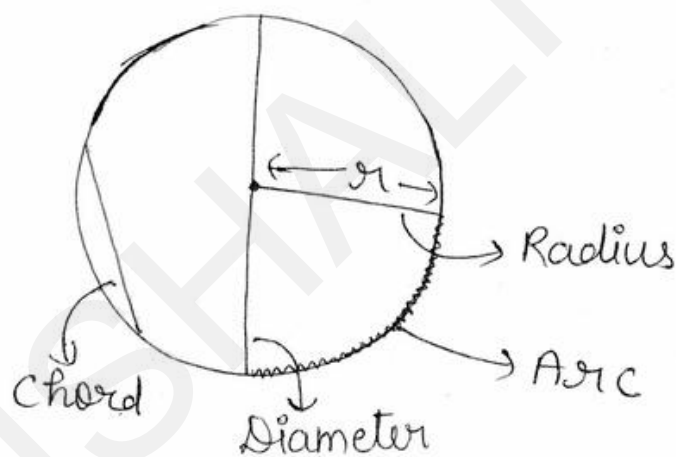
$$\text{Area} = \frac{1}{2} \times d_1 \times d_2$$



Polygon

- Sum of Interior angle = $(n-2) \times 180^\circ$
- Each angle of regular polygon = $\frac{(n-2) \times 180^\circ}{n}$
- Sum of Exterior angle = 360°
- Each Exterior angle of regular polygon = $\frac{360^\circ}{n}$
- Number of diagonals = $\frac{n(n-3)}{2}$

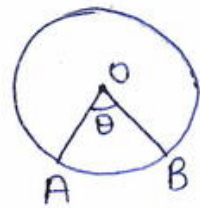
Circle



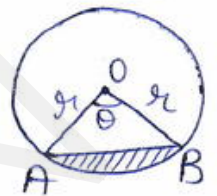
- ★ Area of circle = πr^2
- ★ Circumference = $2\pi r$
- ★ Diameter = $2r$

* Area of sector = $\frac{\pi r^2}{360} \times \theta$

* Length of arc AB = $\frac{2\pi r}{360} \times \theta$



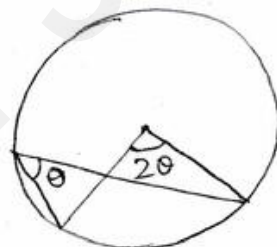
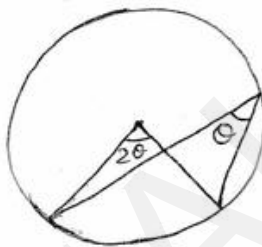
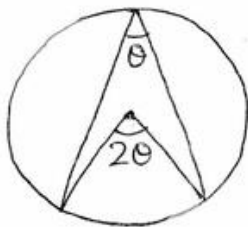
* Area of segment = $\frac{\pi r^2}{360} \times \theta - \frac{1}{2} r^2 \sin \theta$



* Perimeter of Semicircle = $\pi r + 2r$

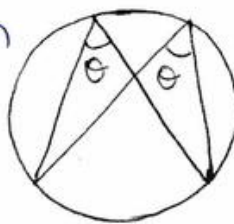
Some important points:-

1.

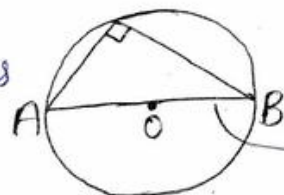


Angle on centre = 2 (Angle on circumference)

2. Angle made by an arc on same side of circle are equal.

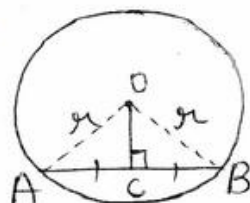


3. Angle made in semi-circle is right angle.



Diameter

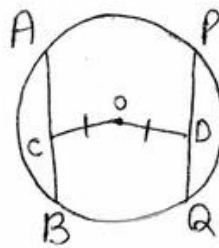
4. A perpendicular drawn from the centre of a circle to a chord bisects the chord.



$OC \perp AB$

$AC = BC$

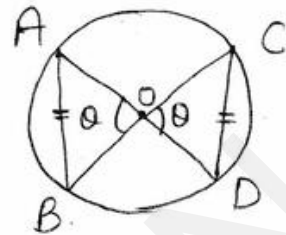
5. If $AB = PQ$
then, $OC = OD$



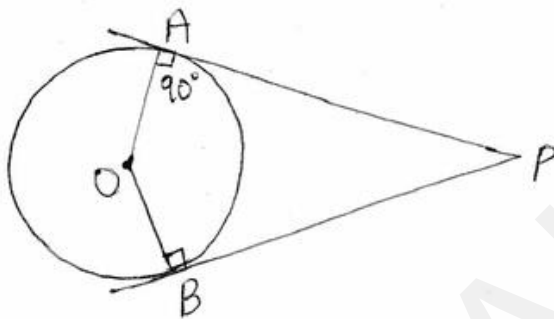
6. Equal chords make equal angle at the centre.

If $AB = CD$, then $\angle AOB = \angle COD$

If $\angle AOB = \angle COD$, then $AB = CD$



7.

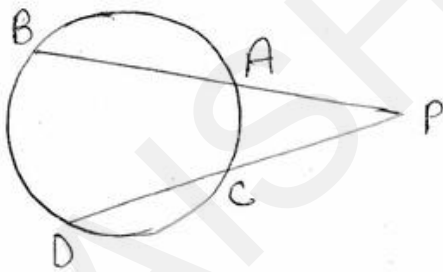


$$PA = PB$$

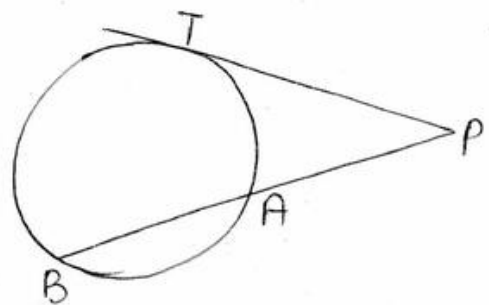
$$\angle PAO = \angle PBO = 90^\circ$$

$$\angle AOB + \angle APB = 180^\circ$$

8.

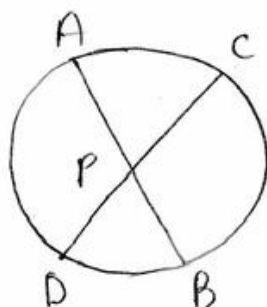


$$PA \times PB = PC \times PD$$



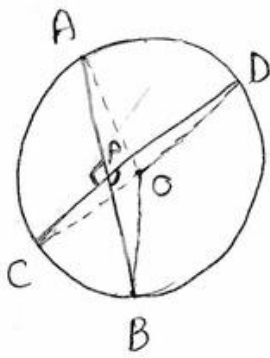
$$PT^2 = PA \times PB$$

9.



$$PA \times PB = PC \times PD$$

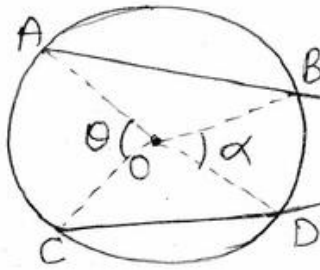
10.



AB and CD are two chords cuts each other internally at Point P, then,

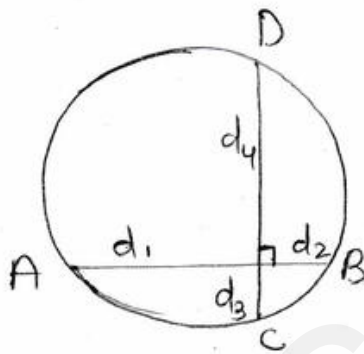
$$\angle APD = \angle BPC = \frac{\angle AOD + \angle BOC}{2}$$

$$\angle APC = 180^\circ - \angle APD$$



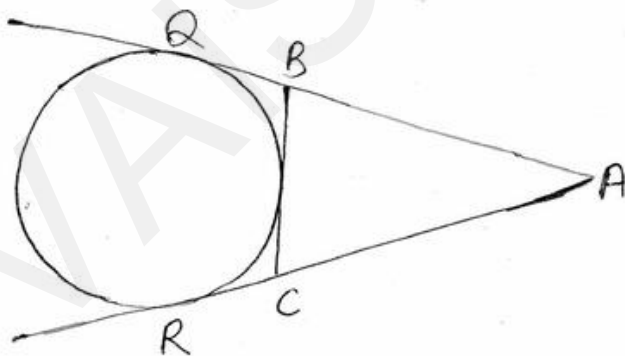
$$\angle APC = \angle BPD = \frac{\theta - \alpha}{2}$$

11.



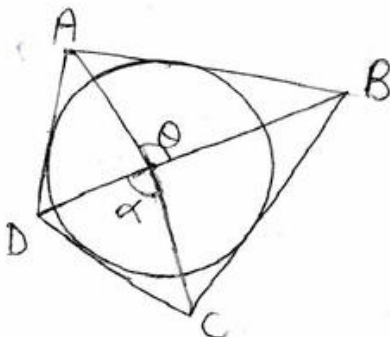
$$\text{diameter} = \sqrt{d_1^2 + d_2^2 + d_3^2 + d_4^2}$$

12.



$$AQ = \frac{\text{Perimeter of } ABC}{2}$$

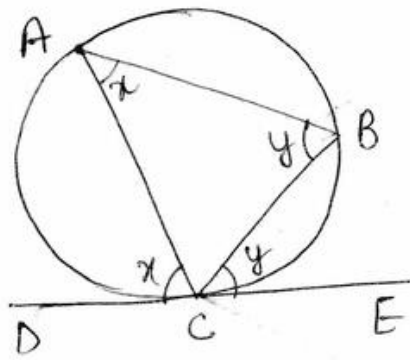
13.



$$\theta + \alpha = 180^\circ$$

$$AB + CD = AD + BC$$

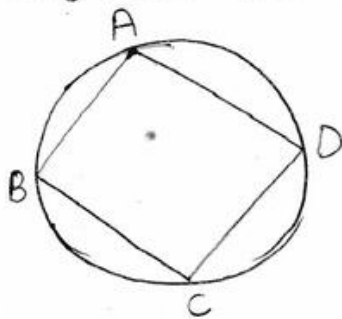
• Alternate Segment Theorem:-



$$\angle BAC = \angle BCE$$

$$\angle ABC = \angle ACD$$

• Cyclic Quadrilateral:-



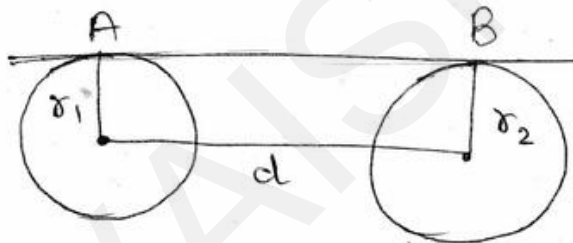
$$\angle A + \angle C = 180^\circ$$

$$\angle B + \angle D = 180^\circ$$

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$s = \frac{a+b+c+d}{2}$$

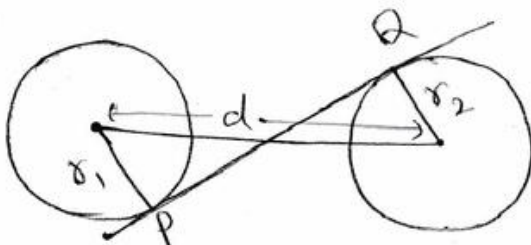
• Common tangent:-



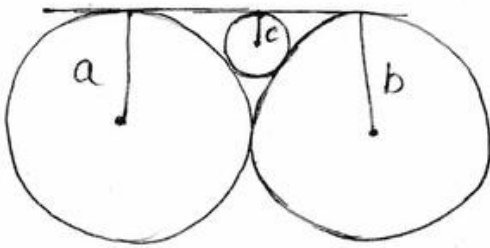
$$AB = \sqrt{d^2 - (r_1 - r_2)^2}$$



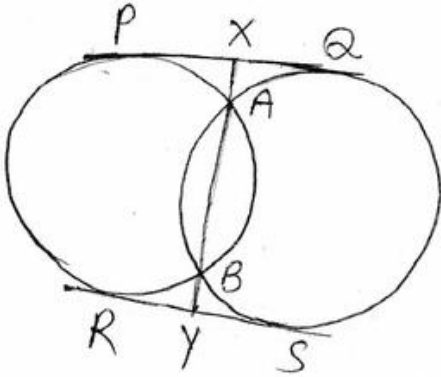
$$AB = 2\sqrt{r_1 r_2}$$



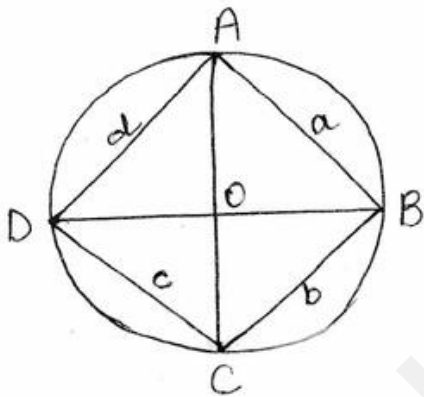
$$PQ = \sqrt{d^2 - (r_1 + r_2)^2}$$



$$\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$$



$$PQ = \sqrt{(XY)^2 - AB^2}$$



$$\frac{ab}{cd} = \frac{OB}{OD}$$

If $OB = OD$
then, $ab = cd$

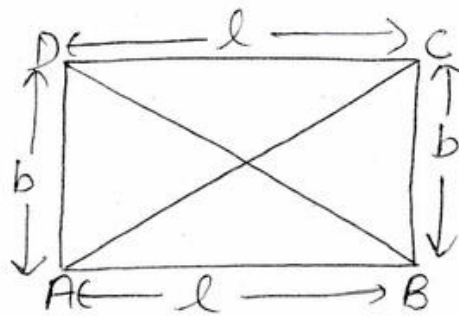
Mensuration 2D

• Rectangle :-

$$\text{Area} = l \times b$$

$$\text{Perimeter} = 2(l + b)$$

$$\text{Diagonal} = \sqrt{l^2 + b^2}$$

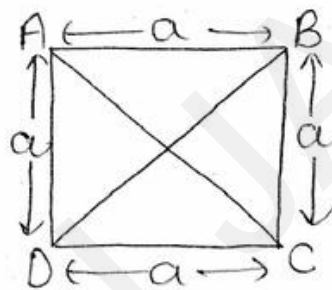


• Square :-

$$\text{Area} = a^2$$

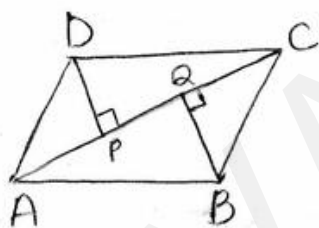
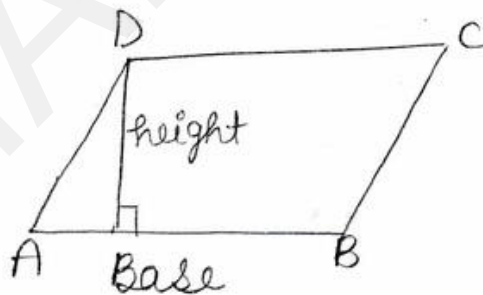
$$\text{Perimeter} = 4a$$

$$\text{Diagonal} = \sqrt{2} \times a$$



• Parallelogram :-

$$\text{Area} = \text{base} \times \text{height}$$



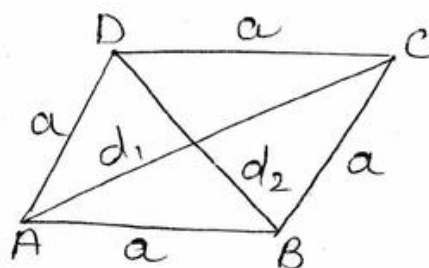
$$\text{Area} = \frac{1}{2} \times AC \times (DP + BQ)$$

• Rhombus :-

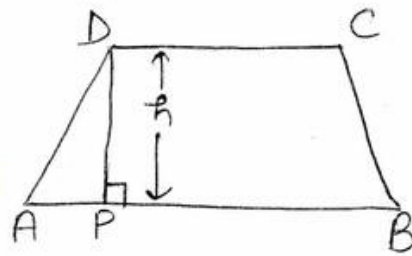
$$(d_1)^2 + (d_2)^2 = 4a^2$$

$$\text{Perimeter} = 4 \times a$$

$$\text{Area} = \frac{1}{2} \times d_1 \times d_2$$



• Trapezium :-

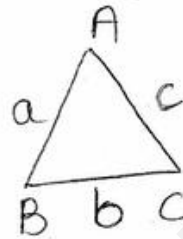


$$\text{Perimeter} = AB + BC + CD + DA$$

$$\text{Area} = \frac{1}{2} \times h \times (AB + CD)$$

$$(DP)^2 = (AP)^2 + (AD)^2$$

• Scalene Triangle :-

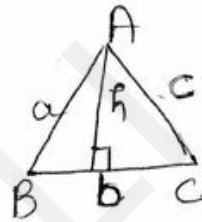


$$\text{Perimeter} = a + b + c$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where, } s = \frac{a+b+c}{2}$$

$$\text{Area} = \frac{1}{2} \times b \times h$$

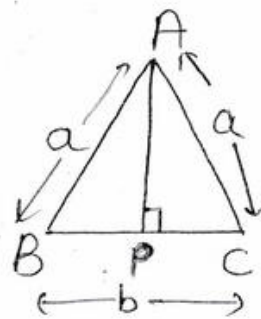


• Isosceles Triangle :-

$$\text{Perimeter} = a + a + b$$

$$\text{Area} = \frac{b}{4} \sqrt{4a^2 - b^2}$$

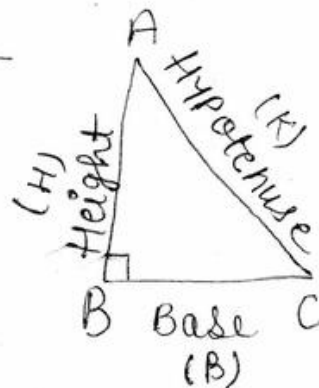
$$\text{Altitude (AP)} = \frac{\sqrt{4a^2 - b^2}}{2}$$



• Right - Angle Triangle :-

$$(K)^2 = (H)^2 + (B)^2$$

$$\text{Area} = \frac{1}{2} \times (B) \times (H)$$

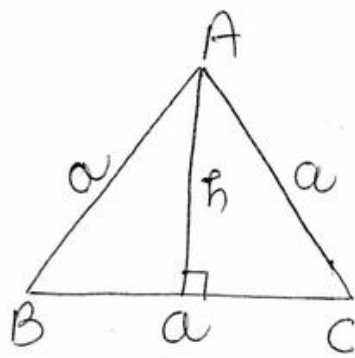


- Equilateral Triangle :-

$$\text{Perimeter} = 3a$$

$$\text{Area} = \frac{\sqrt{3}}{4} a^2$$

$$\text{Height (h)} = \frac{\sqrt{3}}{2} \times a$$

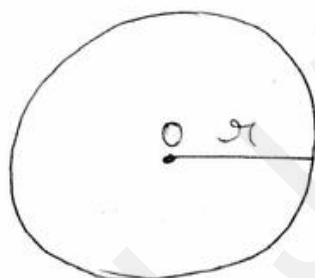


- Circle :-

$$\text{Diameter} = 2r$$

$$\text{Circumference} = 2\pi r$$

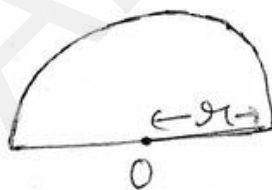
$$\text{Area} = \pi r^2$$



- Semi-circle :-

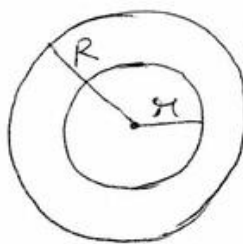
$$\text{Area} = \frac{1}{2} \pi r^2$$

$$\text{Perimeter} = r(\pi + 2)$$

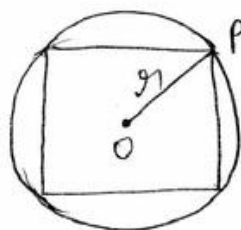


- Ring :-

$$\text{Area} = \pi(R^2 - r^2)$$

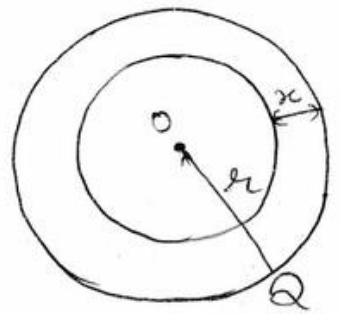


- If the largest square is formed inside a circle, then the area of remaining part except that square = $\frac{8}{7} \times r^2$



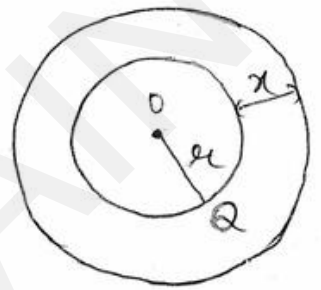
- If there is a path of x unit width inside a circular field, then area of that path

$$= \pi \times (2r - x)$$



- If there is a path of x unit width outside a circular field, then area of that path,

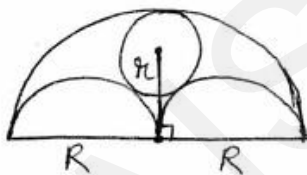
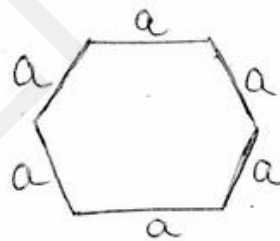
$$= \pi \times (2r + x)$$



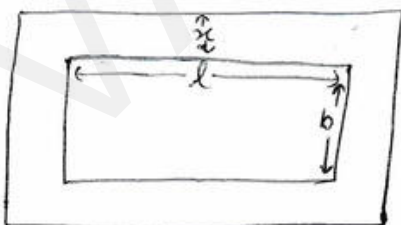
- Hexagon:-

$$\text{Area} = 6 \times \frac{\sqrt{3}}{4} a^2$$

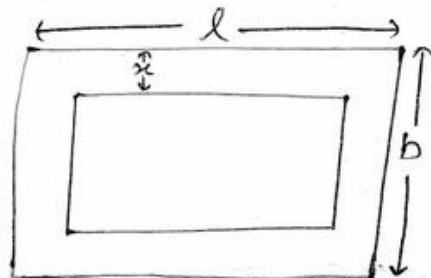
$$\text{Perimeter} = 6a$$



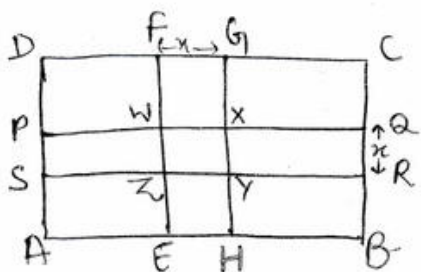
$$r = \frac{R}{3}$$



$$\text{Area of path} = 2x(l + b + 2x)$$



$$\text{Area of Path} = 2x(l + b - 2x)$$



$$\begin{aligned} \text{Area of the path} &= \text{Area of } \square FGHE \\ &+ \text{Area of Rectangle PQRS} - \text{Area of Square WXYZ} \end{aligned}$$

✓

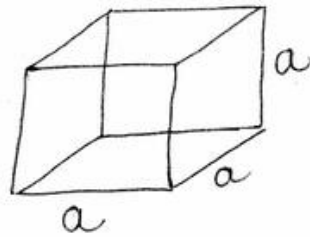
Mensuration 3D

- Cube :-

$$\text{Volume} = a^3$$

$$\text{Total Surface Area} = 6 \times a^2$$

$$\text{Diagonal} = \sqrt{3} \times a$$



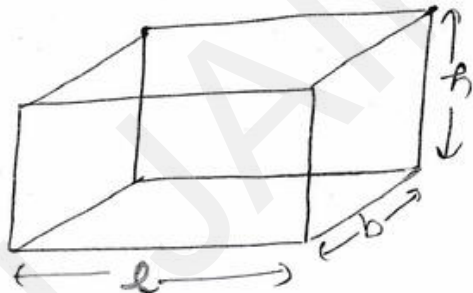
- Cuboid :-

$$\text{Volume} = l \times b \times h$$

$$\text{Total Surface Area} = 2(lb + bh + hl)$$

$$\text{Area of four walls} = 2(l+b)h$$

$$\text{Diagonal} = \sqrt{l^2 + b^2 + h^2}$$



- Cylinder :-

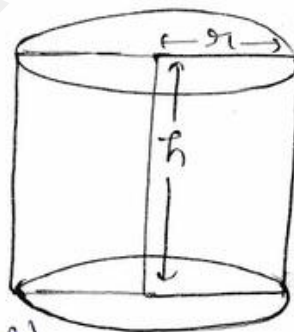
Curved Surface

$$\text{area} = 2\pi r h$$

$$\text{Total Surface Area} = 2\pi r(r+h)$$

$$\text{Volume} = \pi r^2 h$$

$$\text{Volume of hollow cylinder} = \pi h(R^2 - r^2)$$



- Cone :-

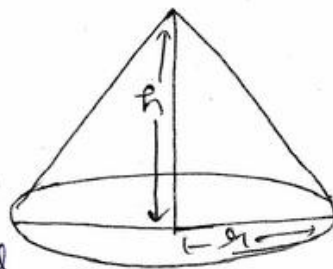
$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

lateral height

$$l = \sqrt{r^2 + h^2}$$

$$\text{Curved Surface Area} = \pi r l$$

$$\text{Total Surface Area} = \pi r(r+l)$$



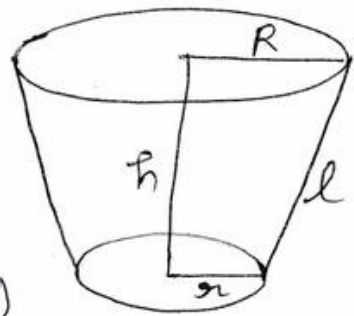
- Frustrum of Cone:-

$$\text{Volume} = \frac{1}{3} \pi h (R^2 + r^2 + Rr)$$

$$\text{Curved Surface Area} = \pi l (R + r)$$

$$\text{Total Surface Area} = \pi [(R + r)l + (R^2 + r^2)]$$

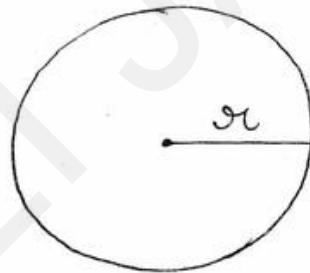
$$\text{lateral height} = \sqrt{h^2 + (R - r)^2}$$



- Sphere:-

$$\text{Total Surface Area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3} \pi r^3$$

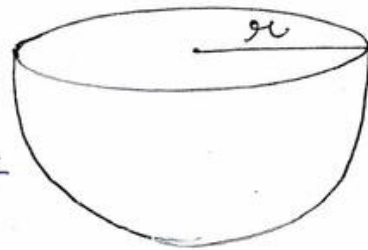


- Hemisphere:-

$$\text{Curved Surface Area} = 2\pi r^2$$

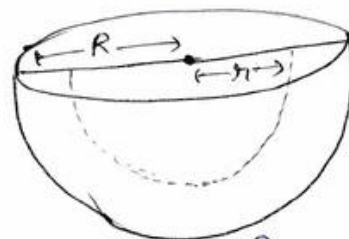
$$\text{Total Surface Area} = 3\pi r^2$$

$$\text{Volume} = \frac{2}{3} \pi r^3$$



- Hollow Sphere:-

$$\text{Volume} = \frac{4}{3} \pi (R^3 - r^3)$$



$$\text{Volume of hollow hemisphere} = \frac{4}{3} \pi (R^3 - r^3)$$

- Pyramid :-

Area = Base area + Sum of areas of all triangular surfaces

Lateral face of a right pyramid = $\frac{1}{2} \times \text{perimeter of Base} \times \text{slant height}$

Volume = $\frac{1}{3} \times \text{Base area} \times \text{height}$

- Prism :-

Total area = Base area + Upper Surface area + Total area of all lateral surface

Volume = Base area $\times$ height

Lateral surface area = Perimeter of Base $\times$ height

Permutation & Combination

Permutation:-

$$\bullet {}^n P_r = \frac{n!}{n-r!}$$

Properties of Permutation:-

- ${}_n P_n = n!$
- ${}_n P_0 = 1$
- ${}_n P_1 = n$
- ${}_n P_{(n-1)} = n!$
- $\frac{{}_n P_r}{{}_n P_{(r-1)}} = n-r+1$
- The number of permutations of n different objects taken ' r ' at a time where repetition is allowed $= n^r$
- The number of permutations of n objects, where p objects are of same kind and rest are all different $= \frac{n!}{p!}$

Combination:-

$${}_n C_r = \frac{n!}{r! (n-r)!}$$

Properties of combination:-

- ${}^nC_0 = {}^nC_n = 1$
- ${}^nC_1 = n$
- ${}^nC_r = {}^nC_{(n-r)}$
- ${}^nC_r = \frac{{}^nP_r}{r!}$
- ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$

$${}^nC_a = {}^nC_b \Rightarrow a+b \text{ \& } n=a+b$$

Relation between permutation and combination

$${}^nP_r = {}^nC_r \times r! , 0 < r \leq n$$

- Multiplication Principal:- If first operation can be performed in 'm' ways and then second operation can be performed in 'n' ways. Then, the two operations taken together can be performed in 'mn' ways. This can be extended to any finite number of operations.
- Addition Principal:- If an operation can be performed in 'm' ways and another operation, which is independent of the first, can be performed in 'n' ways. Then, either of the two operations can be performed in 'm+n' ways. This can be extended to any finite number of mutually exclusive events.

Factorial

$$n \text{ or } n! = n(n-1)(n-2)(n-3)\dots \times 3 \times 2 \times 1$$

$$0! = 1! = 1$$

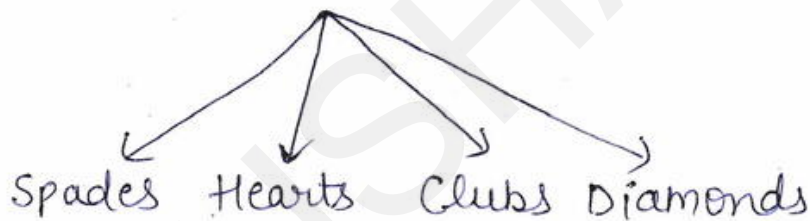
Factorials of negative integers and fractions are not defined.

$$n = n(n-1) = n(n-1)(n-2)$$

Probability

- Probability of a event $P(E) = \frac{\text{favourable outcome}}{\text{Total Possible outcome}}$
- The Probability of each event lies between 0 and 1.
- Sum of all the probabilities of an event is 1.
- Trial
Toss of a coin
Dice is thrown
- All possible outcome
2 [Head & Tail]
6 [1, 2, 3, 4, 5, 6]
- In a deck of cards $\rightarrow$ 52 cards

4 suits (13 card each)



Black colour $\rightarrow$ clubs and Spades

Red colour $\rightarrow$ Heart and Diamonds

Cards in each suits (13) $\rightarrow$ 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace

face cards $\rightarrow$ King, Queen, Jack

Total face Cards $= 3 \times 4 = 12$

- Probability of an event E + Probability of the event not $E = 1$
- The probability of an event that can not happen is 0. Such an event is called impossible event.
- The probability of an event that is certain to happen is 1. Such an event is called sure event.

Statistics

* Mean:-

$$\text{Mean of observations} = \frac{\text{Sum of observations}}{\text{Total no. of observations}}$$

* Median:-

Median is the middle no., when data is arranged in ascending order

- If number of observations is even

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ term} + \left[\left(\frac{n}{2}\right) + 1\right]^{\text{th}} \text{ term}}{2}$$

- If number of observations is odd

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term}$$

Median formula for grouped data

$$L_m + \left[\frac{\frac{n}{2} - f}{f_m} \right] i$$

$n \rightarrow$ Total frequency

$f \rightarrow$ Cumulative frequency of class before the median class.

$f_m \rightarrow$ frequency of the class median

$i \rightarrow$ class width

$L_m \rightarrow$ lower boundary of the class median.

* Mode :-

Mode is the most frequently occurring value in the list.

Mode for grouped data :-

$$= L + h \cdot \frac{f_m - f_1}{(f_m - f_1) - (f_m - f_2)}$$

L = Lower limit of the modal class

h = size of the class interval

f_m = Frequency of the modal

f_1 = frequency of the class preceding the modal class.

f_2 = frequency of the class succeeding the modal class.

* Range :-

Range is the difference between the largest number and smallest number of data

* Mean deviation :-

The average deviation from the mean value of the given data set.

* Variance :-

Variance is the expected value of the squared variation of a random variable from its mean value. Or Variance is the measure of how data points differ from the mean.

Ex:-

	3	4	5	5	8	9	9	9	13	15	Mean=0
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
Deviation from mean	5	4	3	3	0	1	1	1	5	7	
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
	25	16	9	9	0	1	1	1	25	49	

$$\text{Sum} = 136$$

$$\text{Variance} = \frac{136}{10} = 13.6$$

* Standard deviation :-

The standard deviation is a measure of how spread out numbers are.

$$\text{Standard deviation} = \sqrt{\text{Variance}}$$

* Frequency :-

The frequency of a particular value is the number of times the value occurs in the data.